Extraction of the Deuteron Elastic Form Factors from Double Polarization Observables in *ed* Elastic Scattering

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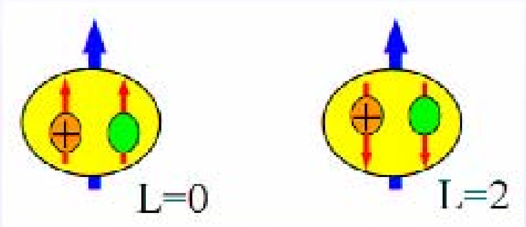
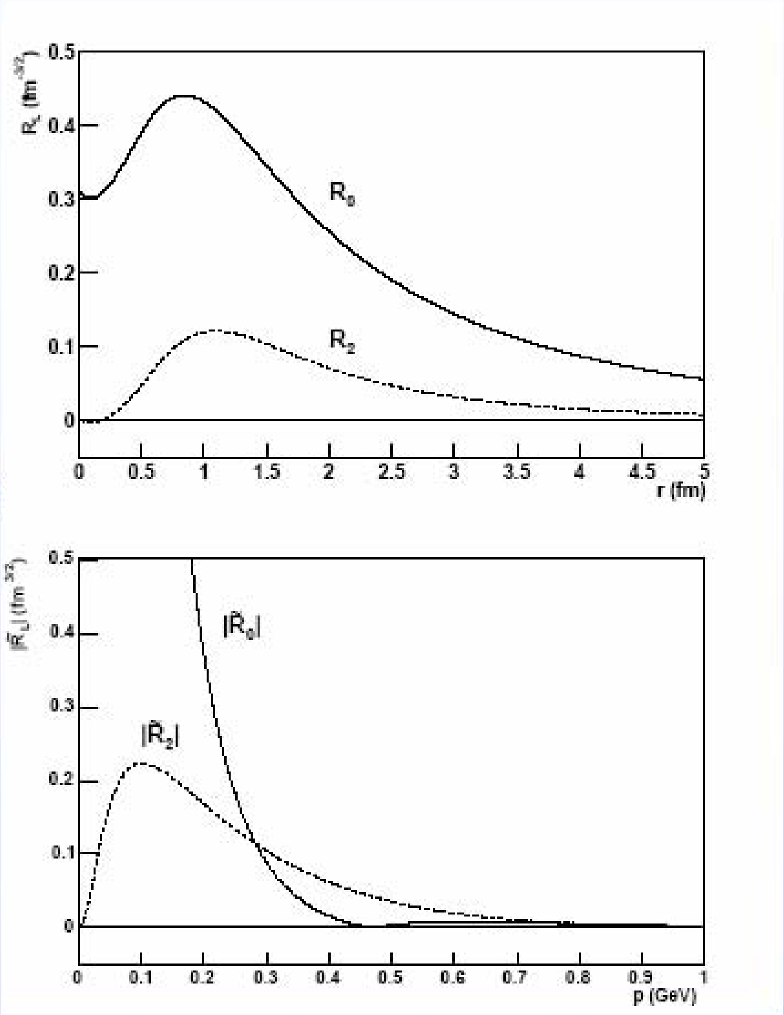
Outline

- *Physics of the Deuteron Wave Functions*
- *Sensitivity to Potentials*
- *Deuteron Elastic Form Factors and Rosenbluth Separation*
- *Polarization Observables*
- *Experimental Asymmetries*
- *Sensitivity to Ingredients of the Models*
- *Extraction of the Form Factors*

Deuteron Wave Function





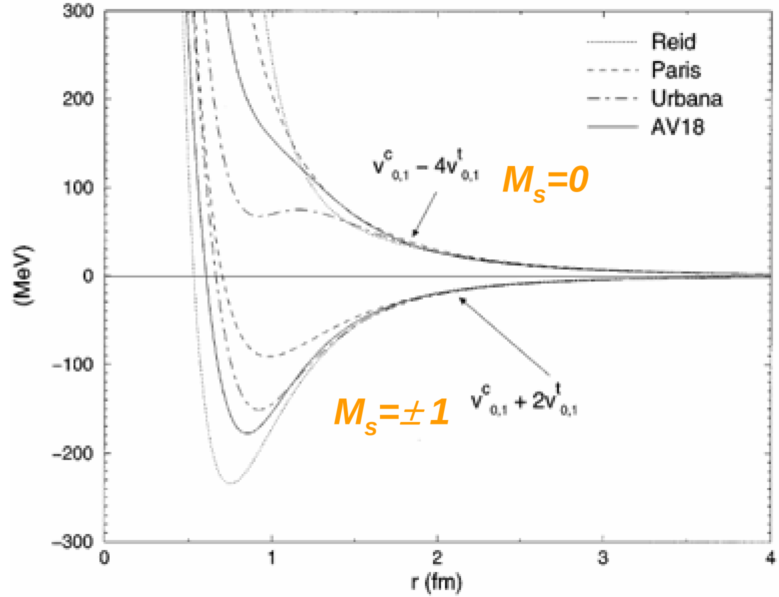
- Tensor force in NN interaction
  $\rightarrow$ L=0,2 admixture

$$\psi^{\pm,0}(\vec{r}) = R_0(r)\, Y_{110}^{\pm,0}(\Omega_r) + R_2(r)\, Y_{112}^{\pm,0}(\Omega_r)$$

- Spin-dependent momentum distr.
  L=2 dominates for $p > 0.3$ GeV/c
- $R_L(p)$ probed by $\vec{\mathrm{d}}(\vec{\mathrm{e}},\mathrm{e'p})$ tensor and beam-vector asymmetries
- Integrals probed in ed elastic

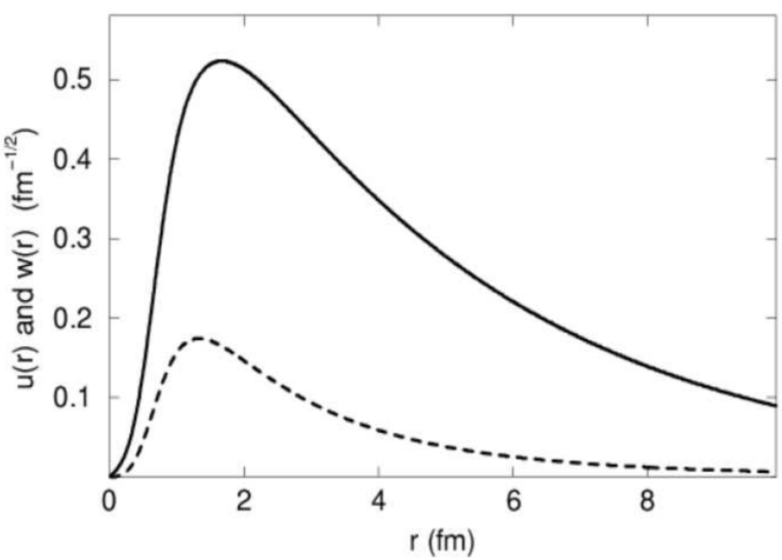
NN Potential & Deuteron Wave Function

- ***NN* Potential models**
  fit to *NN*-scattering data



- **Deuteron as THE *NN* bound state**

NRIA:
  Schrödinger equation under *NN* potential
- **Tensor Force → *D*-wave →** G_Q



Polarized Wave Functions

- **Wave functions M=0, ±1 states:**

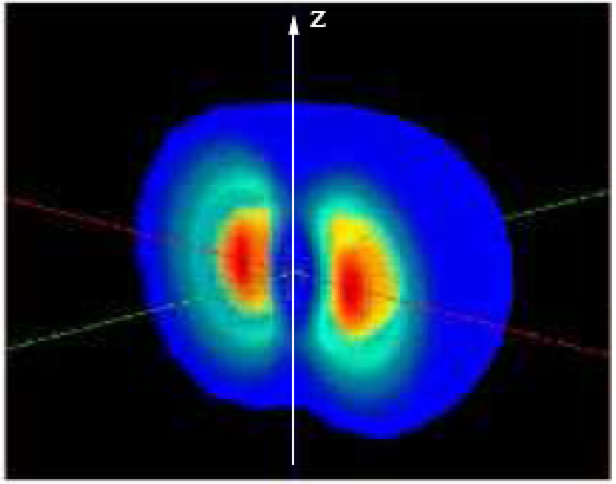
  coupling of spherical and spin harmonics

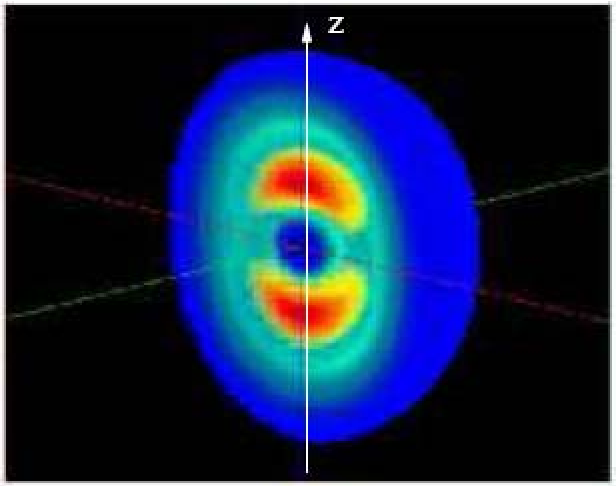
$$\Psi_d^0(\mathbf{r}) = \sqrt{\frac{4}{\pi}}\left[\frac{u(r)}{r} - \sqrt{2}\frac{w(r)}{r}\left(\frac{3}{2}\cos^2\theta - \frac{1}{2}\right)\right]|1,0\rangle$$
$$+ \sqrt{\frac{9}{\pi}}\frac{w(r)}{r}\sin\theta\cos\theta\left[e^{-i\phi}|1,+\rangle - e^{i\phi}|1,-\rangle\right],$$
$$\Psi_d^{\pm1}(\mathbf{r}) = \sqrt{\frac{4}{\pi}}\left[\frac{u(r)}{r} + \sqrt{\frac{1}{2}}\frac{w(r)}{r}\left(\frac{3}{2}\cos^2\theta - \frac{1}{2}\right)\right]|1,\pm\rangle$$
$$\pm \sqrt{\frac{9}{\pi}}\frac{w(r)}{r}e^{\pm i\phi}\sin\theta\cos\theta\,|1,0\rangle + \sqrt{\frac{9}{2\pi}}\frac{w(r)}{r}e^{\pm 2i\phi}\sin^2\theta\,|1,\pm\rangle\,.$$

$R_0(r) \rightarrow u(r)/r$

$R_2(r) \rightarrow w(r)/r$

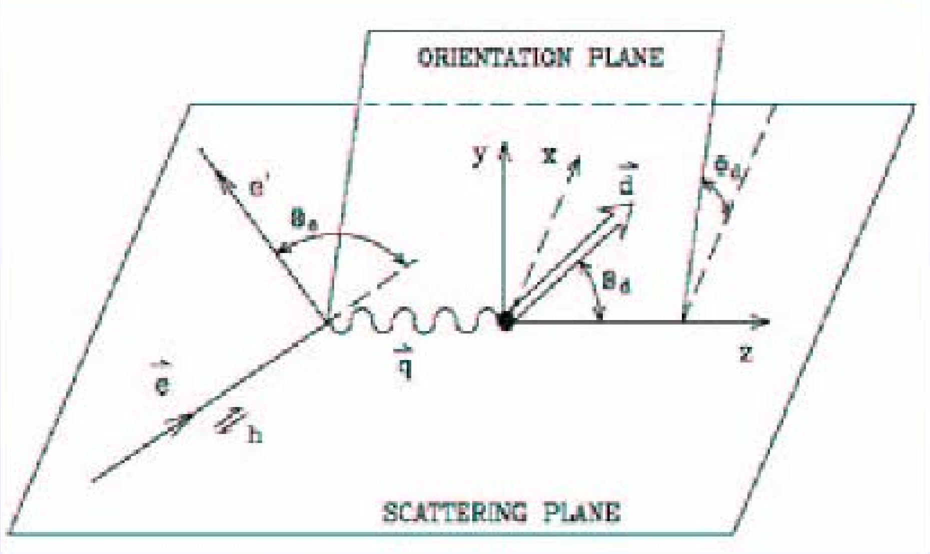
- **The donut and the dumb bell:**





Elastic Electron-Deuteron Scattering

- S=1 $\longrightarrow$ 3 elastic form factors G_C, G_Q, G_M
- G_Q arises from tensor force/D-state



- Non-relativistic deuteron without MEC:

$$G_C = G_E^s D_C \qquad D_C(Q^2) = \int_0^\infty dr\, r^2 \left(R_0(r)^2 + R_2(r)^2\right) j_0(Qr/2)$$

$$G_Q = G_E^s D_Q \qquad D_Q(Q^2) = \frac{1}{\sqrt{2}\eta} \int_0^\infty dr\, r^4 R_2(r) \left(R_0(r) - \frac{R_2(r)}{\sqrt{8}} \right) j_2(Qr/2)$$

$$G_M = \frac{m_d}{2m_p} \left(G_M^s D_M + G_E^s D_E\right) \qquad D_M(Q^2) = \int_0^\infty dr\, r^2 \Big[\left(2R_0(r)^2 - R_2(r)^2\right) j_0(Qr/2) + \left(\sqrt{2}R_0(r)R_2(r) + R_2(r)^2\right) j_2(Qr/2) \Big]$$

$$G^s_i = G^p_i + G^n_i \quad i = E,M \qquad D_E(Q^2) = \frac{3}{2} \int_0^\infty dr\, r^2 R_2(r)^2$$

Deuteron Density Functions

Calculate density functions:

$$\rho^{m_d}(\vec{r}') = \Psi^{m_d}(\vec{r})^* \Psi^{m_d}(\vec{r})$$

$$\rho^0(\vec{r}') = \frac{4}{\pi}\left[C_0(r) - 2C_2(r)P_2(\cos\theta)\right]$$

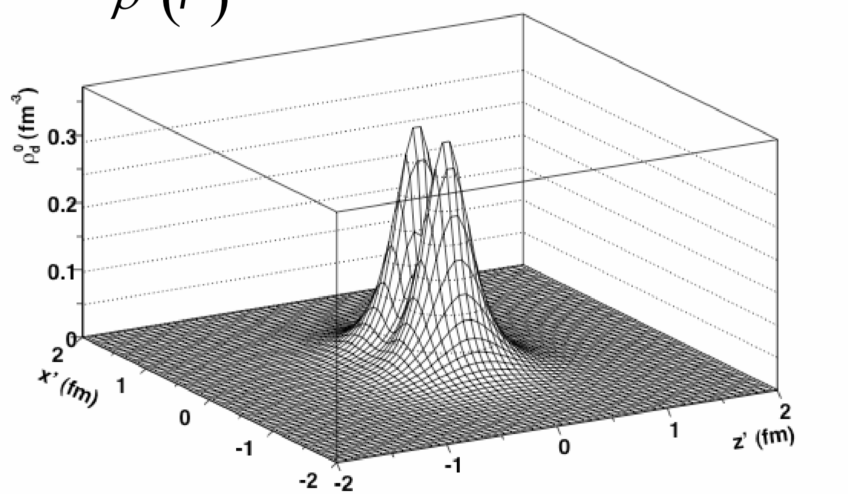
$$\rho^{\pm 1}(\vec{r}') = \frac{4}{\pi}\left[C_0(r) + C_2(r)P_2(\cos\theta)\right]$$

Straightforward form:

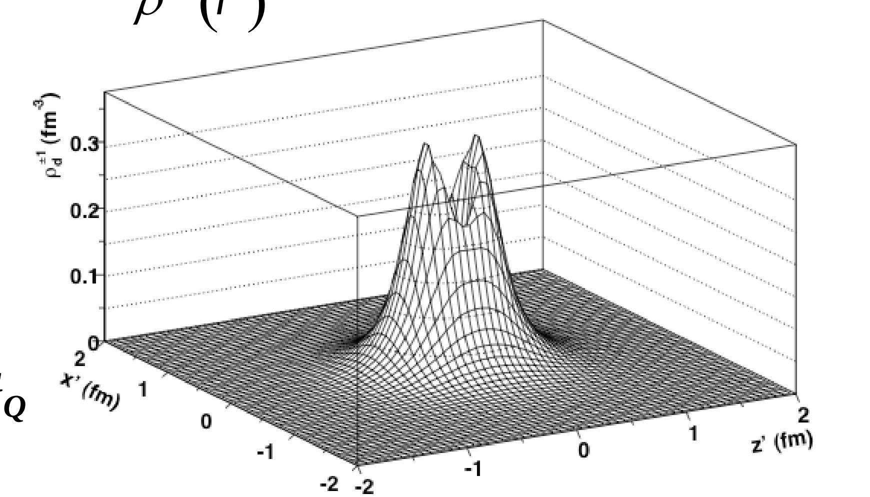
$$C_0(r) \equiv R_0(r)^2 + R_2(r)^2 \quad \rightarrow \boldsymbol{G_C}$$

$$C_2(r) \equiv R_2(r)\left(\sqrt{2}R_0(r) - \frac{1}{2}R_2(r)\right) \quad \rightarrow \boldsymbol{G_Q}$$

$\rho^0(\vec{r}')$



$\rho^{\pm 1}(\vec{r}')$



Electromagnetic Structure

Unpolarized Scattering Cross Section

$$\left(\frac{d\sigma}{d\Omega}\right)_0 = \left(\frac{d\sigma}{d\Omega}\right)_{Mott} \cdot f_{rec}^{-1} \cdot S$$

- $$S = A(Q^2) + B(Q^2) tan^2 \tfrac{\theta_e}{2}, \qquad \tau = \tfrac{Q^2}{4M_d^2}$$
  $$A(Q^2) = G_C^2(Q^2) + \tfrac{8}{9}\tau^2 G_Q^2(Q^2) + \tfrac{2}{3}\tau G_M^2(Q^2)$$
  $$B(Q^2) = \tfrac{4}{3}(1+\tau) G_M^2(Q^2)$$
- **Rosenbluth Separation**: Vary E_{beam} and θ_e at fixed Q^2.
  - can separate A and B, and from B get G_M
  - can <u>not</u> separate G_C and G_Q

We need another observable!

Polarization Observables

Polarized Scattering Cross Section

$$\frac{d\sigma}{d\Omega}(h, P_z, P_{zz}) = \Sigma + h\Delta$$

- $\Sigma = (\frac{d\sigma}{d\Omega})_0[1+\Gamma] \quad \longrightarrow \quad \Gamma$ contains tensor terms T_{2q}
- h=beam helicity, Δ contains vector terms T^e_{1q}
- And we can write $T_{kq} \longrightarrow T_{kq}(G_C, G_Q, G_M)$

What do these look like??

Tensor-Polarized Elastic Scattering*

- Tensor asymmetry and tensor analyzing powers

$$A_d^T = \frac{3}{2}\left(\cos^2\theta_d - 1\right) T_{20} - \sqrt{\frac{3}{2}} \sin 2\theta_d \cos\phi_d T_{21} + \sqrt{\frac{3}{2}} \sin^2\theta_d \cos 2\phi_d T_{22}$$

$$T_{20}(Q^2, \theta_e) = \frac{1}{\sqrt{2}S_0}\left[\frac{8}{3}\eta\, G_C G_Q + \frac{8}{9}\eta^2 G_Q^2 + \frac{1}{3}\eta\left(1 + 2\,(1+\eta)\tan^2\frac{\theta_e}{2}\right) G_M^2\right]$$

$$T_{21}(Q^2, \theta_e) = \frac{1}{\sqrt{3}S_0} 2\eta \sqrt{\eta + \eta^2 \sin^2\frac{\theta_e}{2}} \sec\frac{\theta_e}{2}\, G_M G_Q$$

$$T_{22}(Q^2, \theta_e) = -\frac{1}{2\sqrt{3}S_0}\eta\, G_M^2$$

T_{20} large > T_{21} medium > T_{22} small

$\eta = \tau = Q^2 / 4M^2$

Vector-Polarized Elastic Scattering*

- Beam-vector asymmetry and vector analyzing powers

$$A_{ed}^{V} = \sqrt{3}\left(\frac{1}{\sqrt{2}}\cos\theta_d\, T_{10}^{e} - \sin\theta_d \cos\phi_d\, T_{11}^{e}\right)$$

$$T_{10}^{e}(Q^2,\theta_e) = -\frac{\sqrt{2}}{\sqrt{3}S_0}\eta\sqrt{(1+\eta)\left(1+\eta\sin^2\frac{\theta_e}{2}\right)}\sec\frac{\theta_e}{2}\tan\frac{\theta_e}{2}\, G_M^2$$

$$T_{11}^{e}(Q^2,\theta_e) = \frac{2}{\sqrt{3}S_0}\sqrt{\eta\,(1+\eta)}\tan\frac{\theta_e}{2}\, G_M\left(G_C + \frac{1}{3}\eta G_Q\right)$$

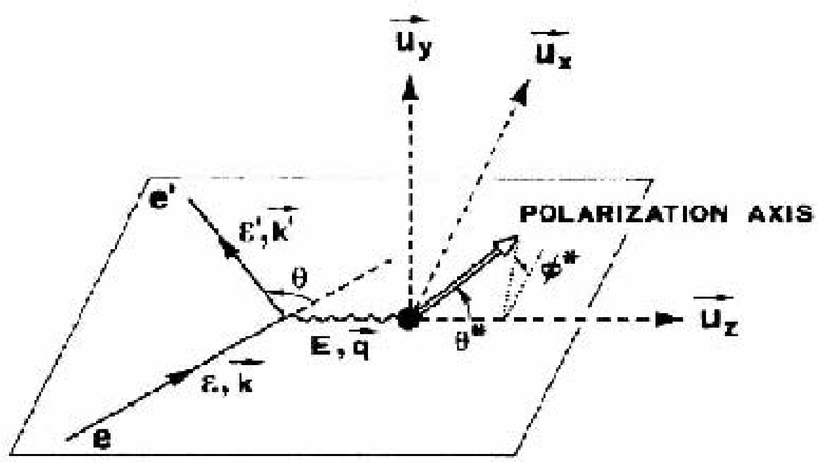
$\boldsymbol{T^e_{11}}$ significant, $\boldsymbol{T^e_{10}}$ not … because $\boldsymbol{G_M^2 << G_C^2}$

Beam-Target Vector Asymmetry

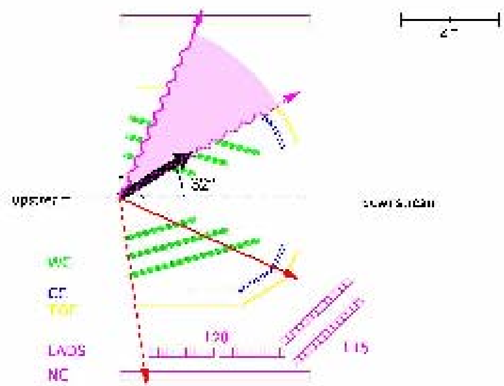
$$A^V_{ed\ theory} \equiv \frac{\Delta}{\Sigma} = \sqrt{3}\,[\,\frac{1}{\sqrt{2}} cos\theta^* T^e_{10}(Q^2,\theta_e)\ -\ sin\theta^* cos\phi^* T_{11}(Q^2,\theta_e)]$$

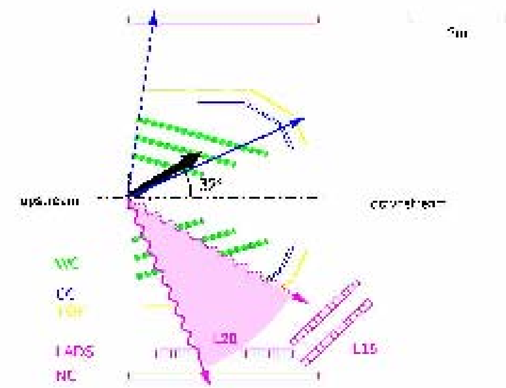
target polarization angles w.r.t. $\vec{q}$ are θ^* and ϕ^*

Scattering & Reaction Planes [2]



Parallel & Perpendicular Kinematics





Parallel Kinematics:
Electron Right
$\vec{q}$ Left & $\sim \parallel \theta_T$

Perpendicular Kinematics:
Electron Left
$\vec{q}$ Right & $\sim \perp \theta_T$

Extracting T_{10}^{e} and T_{11}^{e}

- Exploit the symmetrical geometry of BLAST!
- Measure $A^{V}_{ed,\perp}$ and $A^{V}_{ed,\parallel}$ simultaneously
- Extract the vector analyzing powers T_{10}^{e} and T_{11}^{e}

$$T_{10}^{e} = \sqrt{\frac{2}{3}}\left[\frac{sin\theta^{*}_{\parallel}cos\phi^{*}_{\parallel}A_{\perp} - sin\theta^{*}_{\perp}cos\phi^{*}_{\perp}A_{\parallel}}{cos\phi^{*}_{\parallel}sin\theta^{*}_{\perp}cos\phi^{*}_{\perp} - cos\theta^{*}_{\perp}sin\theta^{*}_{\parallel}cos\phi^{*}_{\parallel}}\right]$$

$$T_{11}^{e} = \frac{\sqrt{3}}{3}\left[\frac{cos\theta^{*}_{\parallel}A_{\perp} - cos\theta^{*}_{\perp}A_{\parallel}}{cos\theta^{*}_{\perp}sin\theta^{*}_{\parallel}cos\phi^{*}_{\parallel} - cos\theta^{*}_{\parallel}sin\theta^{*}_{\perp}cos\phi^{*}_{\perp}}\right]$$

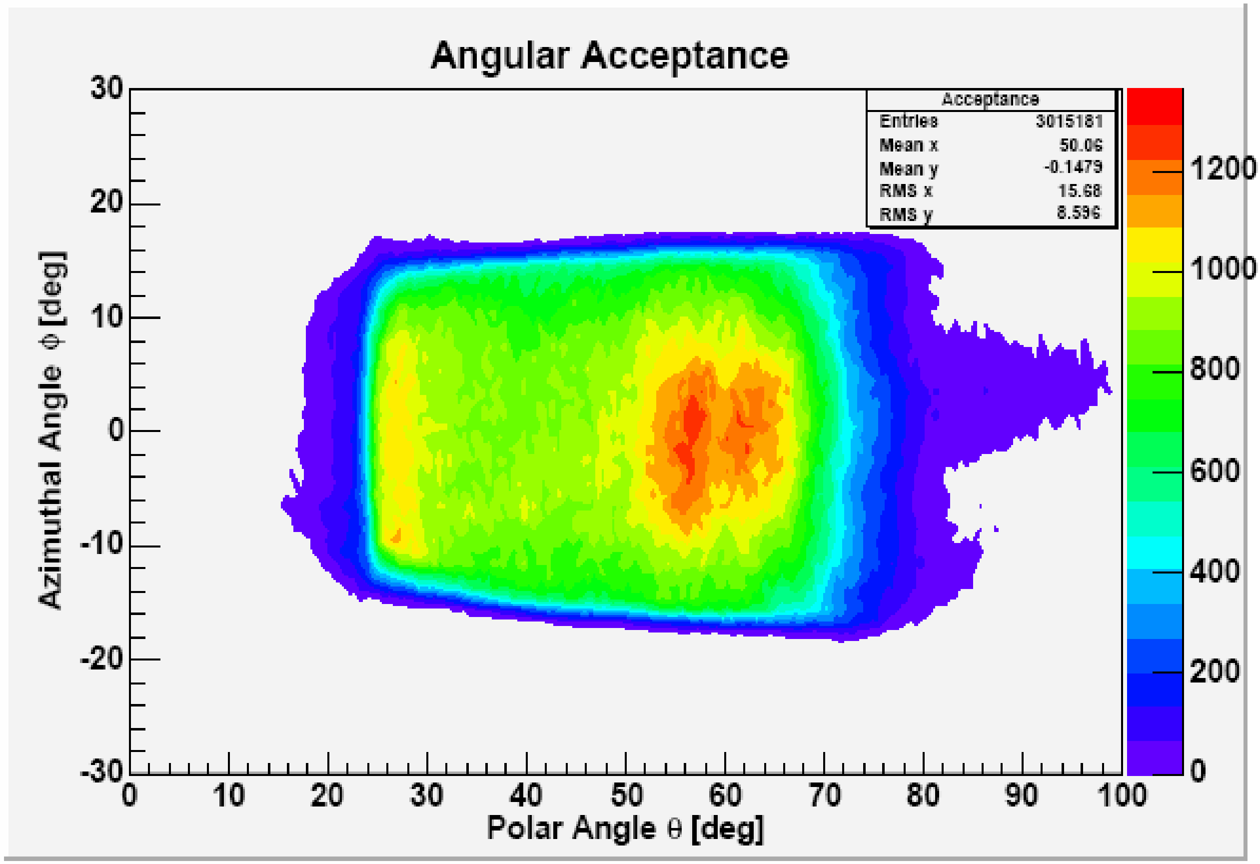
Theory $\Longleftrightarrow$ Experiment

$$A^V_{ed,\ theory} \equiv \frac{\Delta}{\Sigma} = \sqrt{3}\,[\ \frac{1}{\sqrt{2}} cos\theta^*\ \ T^e_{10}(Q^2)\ \ -\ sin\theta^* cos\phi^*\ \ T^e_{11}(Q^2)\ ]$$

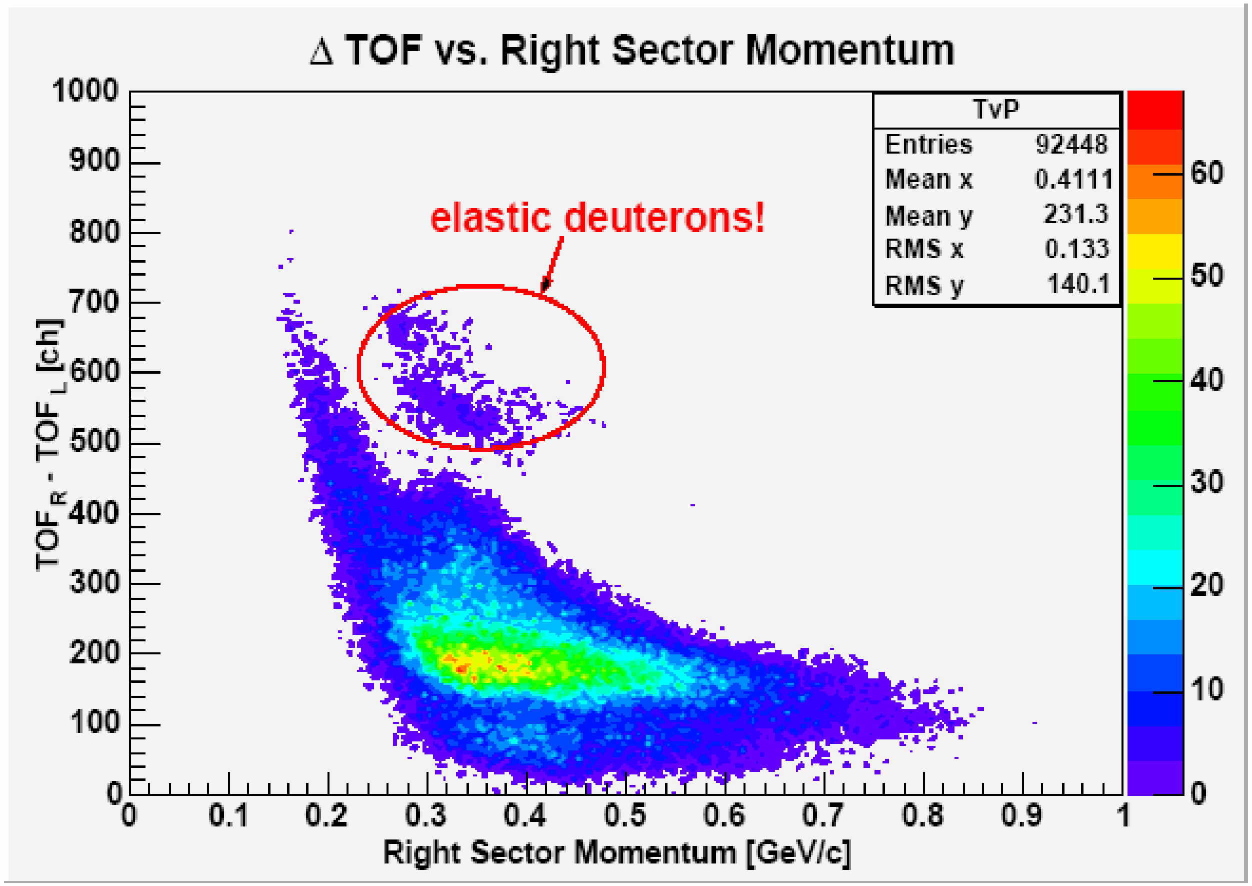
$$A^V_{ed,\ exp} \equiv \frac{1}{4hP_z\sigma_0}[\sigma(+,+,+1) - \sigma(-,+,+1) - \sigma(+,-,+1) + \sigma(-,-,+1)]$$

BEAM-TARGET POLARIZATION STATES $\sigma(h, V, T)$

General Event Selection



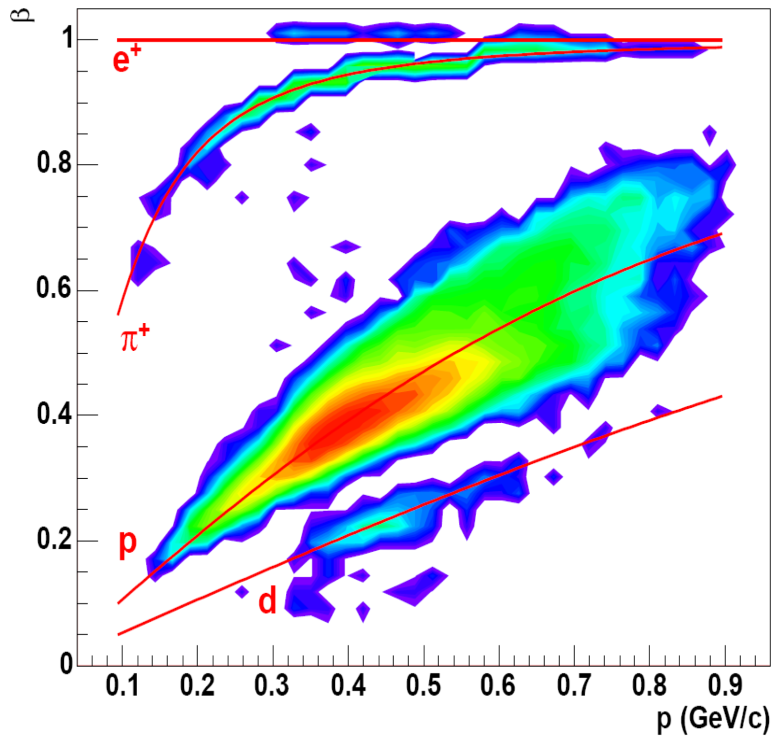
Selection of Elastic Deuterons

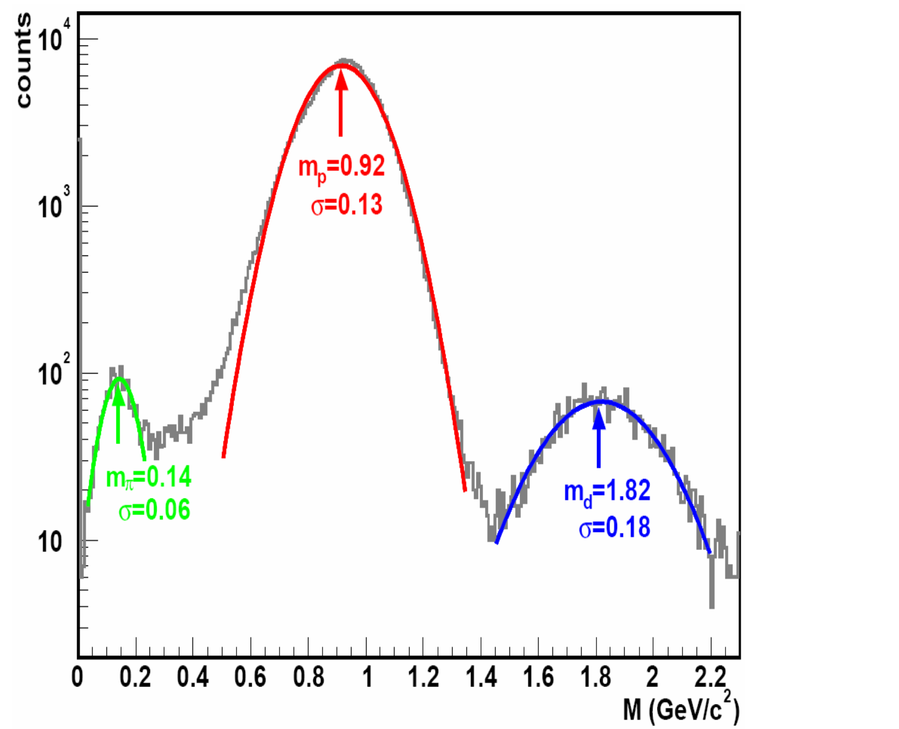


PID: Proton vs. Deuteron

- **TOF, Geometric Trajectory → β = v/c, momentum, β → M**
- **$\sigma_M \cong$ 100 MeV**
- **Cut between proton and deuteron:**

  1.5GeV: 4-5 σ_M separation

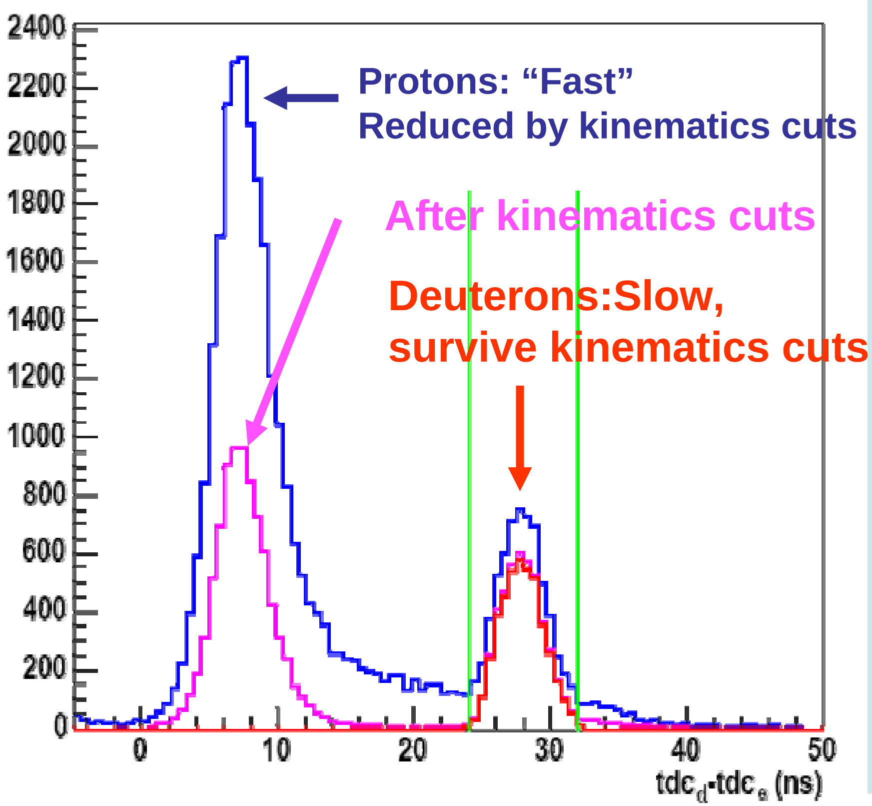


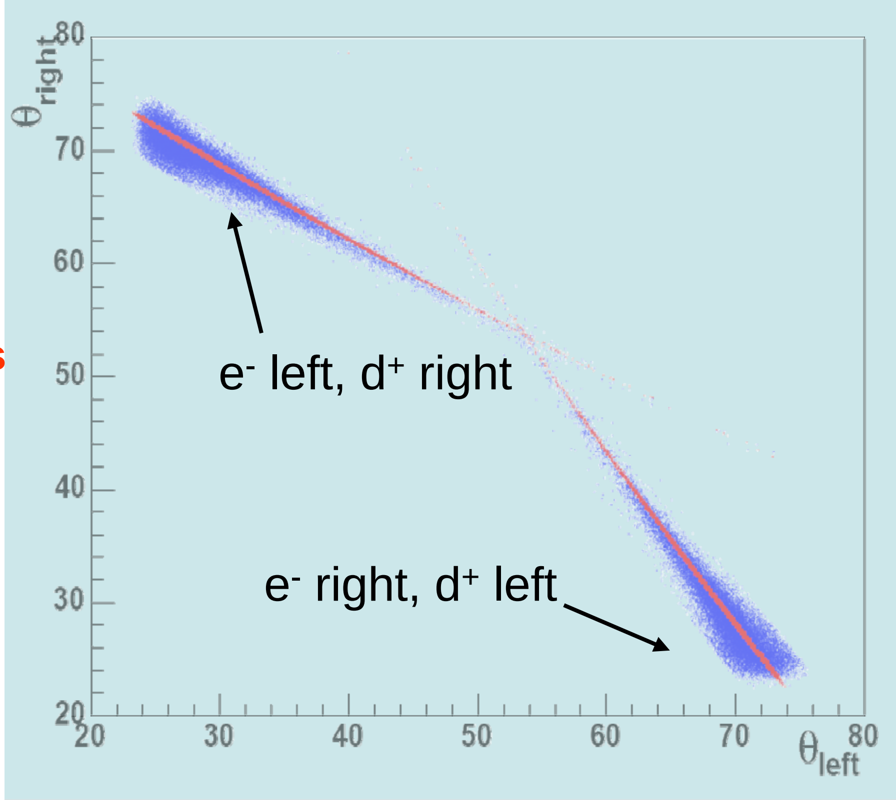


e-d Elastic Event Selection

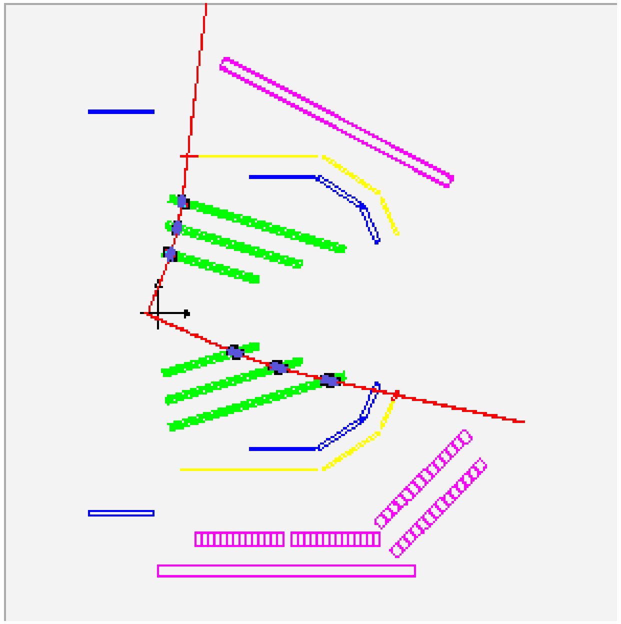
❖ **Timing:**
TOF(p)-TOF(e) ~ 10ns
TOF(d)-TOF(e) ~ 20-30ns

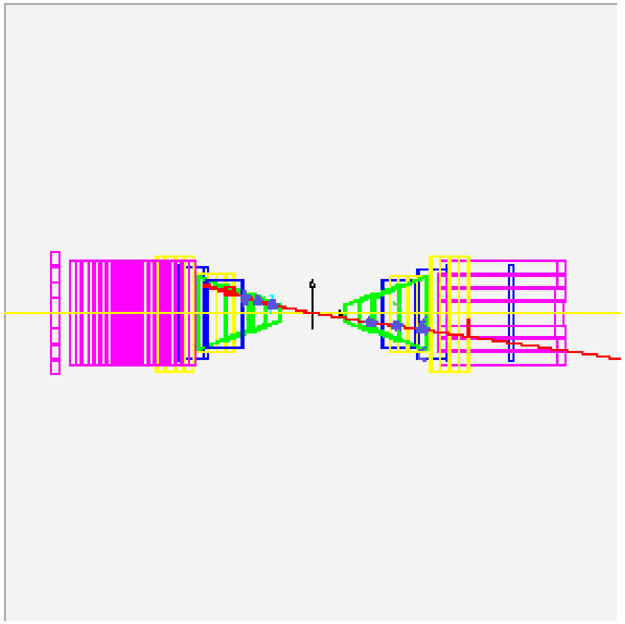
❖ **Kinematics:**
σ_{p_e}=24MeV, σ_{θ_d}=1°, σ_ϕ=1°
… …





Good Elastic Candidate

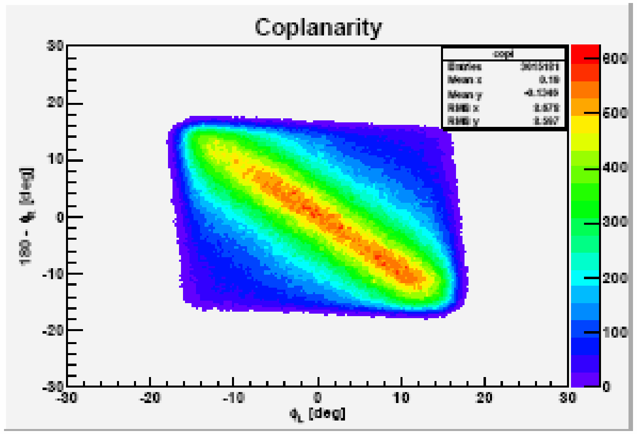




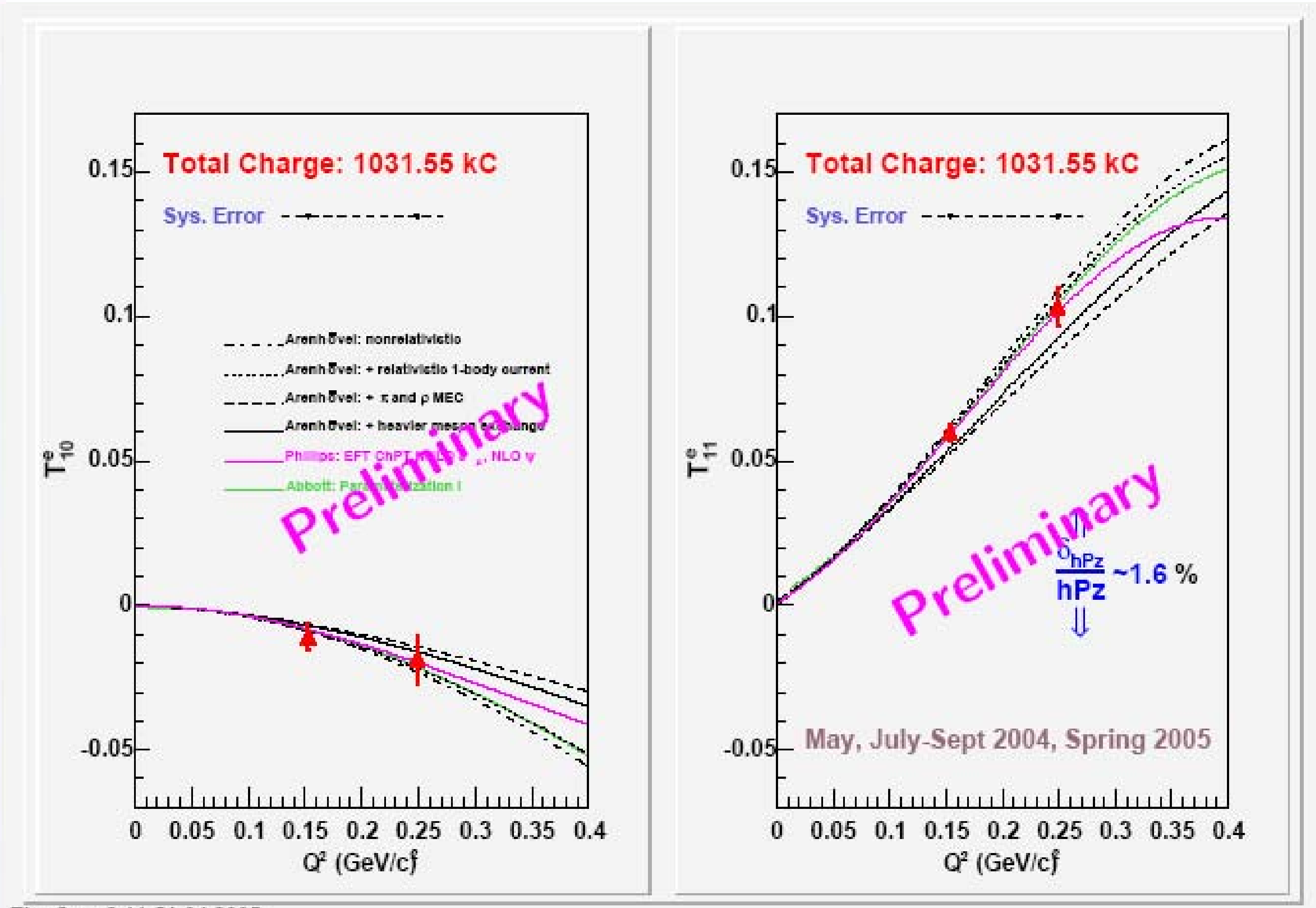
- General data quality cuts satisfied
- Elastic kinematics and timing cuts satisfied

Selection of Elastic Deuterons

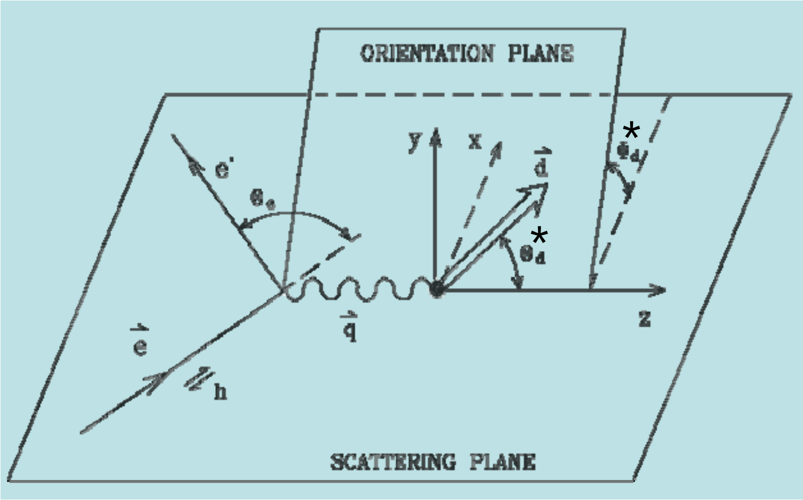
- Two-body final state is coplanar with beam axis
- Cut on coplanarity!



Vector Analyzing Powers



Elastic Electron Deuteron Scattering



- e-d elastic scattering: G_C G_M G_Q
- Rosenbluth Separation

$$A(Q^2) = G_C^2(Q^2) + \frac{8}{9}\eta^2 G_Q^2(Q^2) + \frac{2}{3}\eta G_M^2(Q^2)$$
$$B(Q^2) = \frac{4}{3}\eta(1+\eta)G_M^2(Q^2)$$

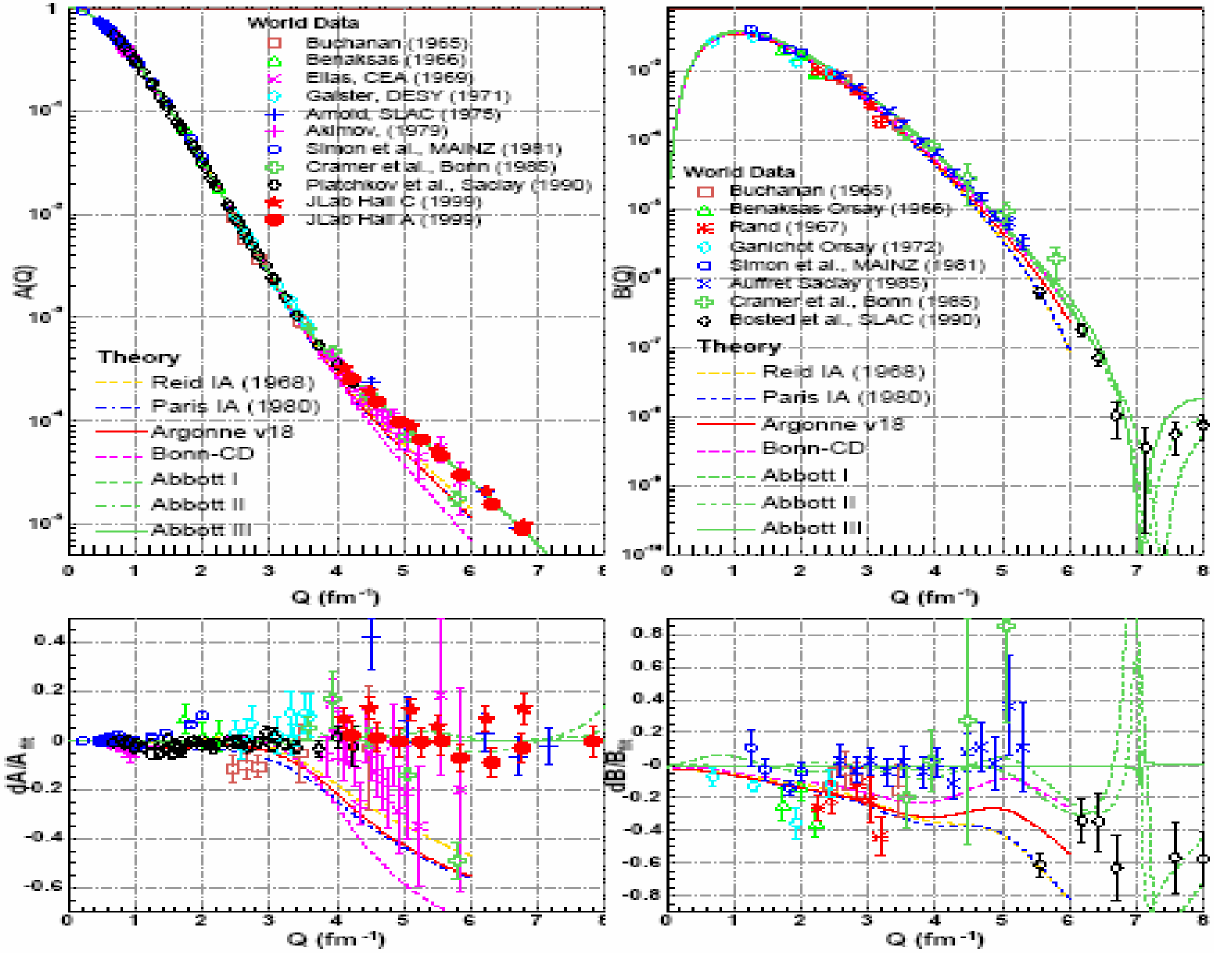
- 3[rd] Measurement to separate 3 form factors

$$T_{20} = -\frac{1}{\sqrt{2}S}\left[\frac{8}{3}\eta G_C G_Q + \frac{8}{9}\eta^2 G_Q^2 + \frac{1}{3}\eta\left[1 + 2(1+\eta)tan^2\frac{\theta_2}{2}\right]G_M^2\right]$$

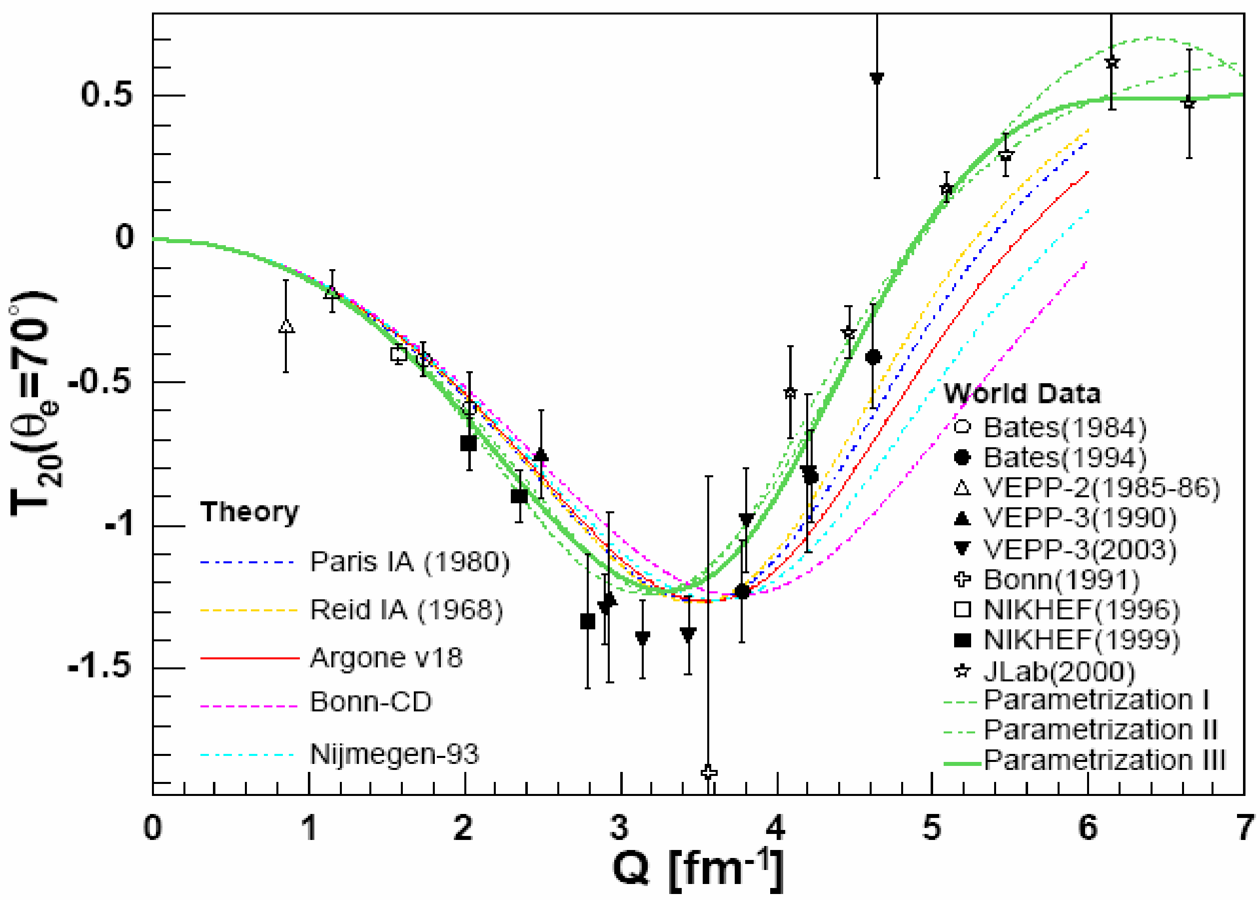
- Tensor Asymmetry in e-d elastic scattering

$$\mathbf{A} = \sqrt{2}\frac{N^+ - N^-}{N^- \cdot P_{zz}^+ - N^+ \cdot P_{zz}^-}$$
$$= \frac{3\cos^2\theta_d^* - 1}{2}T_{20} - \sqrt{\frac{3}{2}}\sin 2\theta_d^* \cos\phi_d^* T_{21} + \sqrt{\frac{3}{2}}\sin^2\theta_d^* \cos 2\phi_d^* T_{22}.$$

Non-relativistic calculations without MEC, RC



Non-relativistic calculations without MEC, RC



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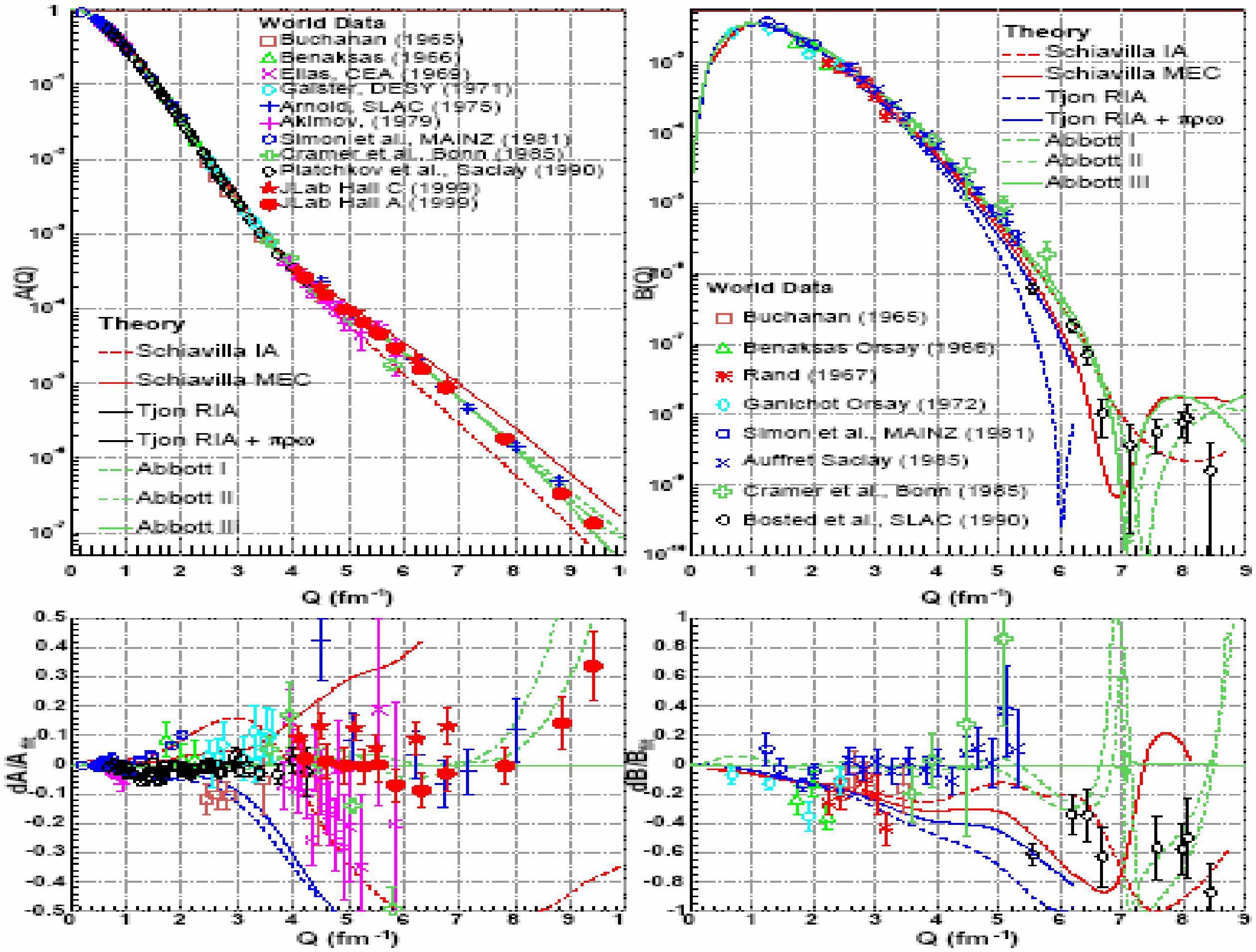
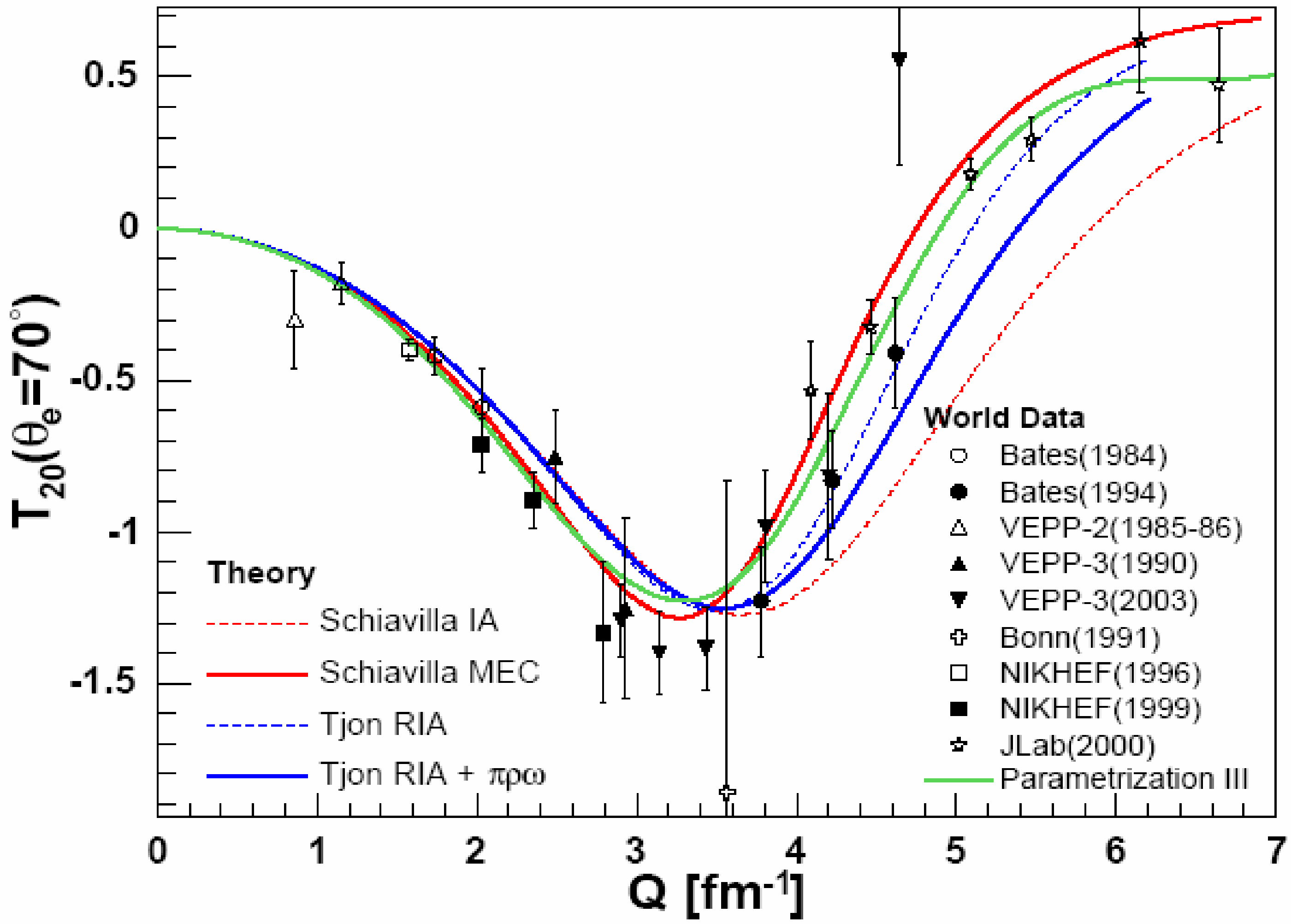
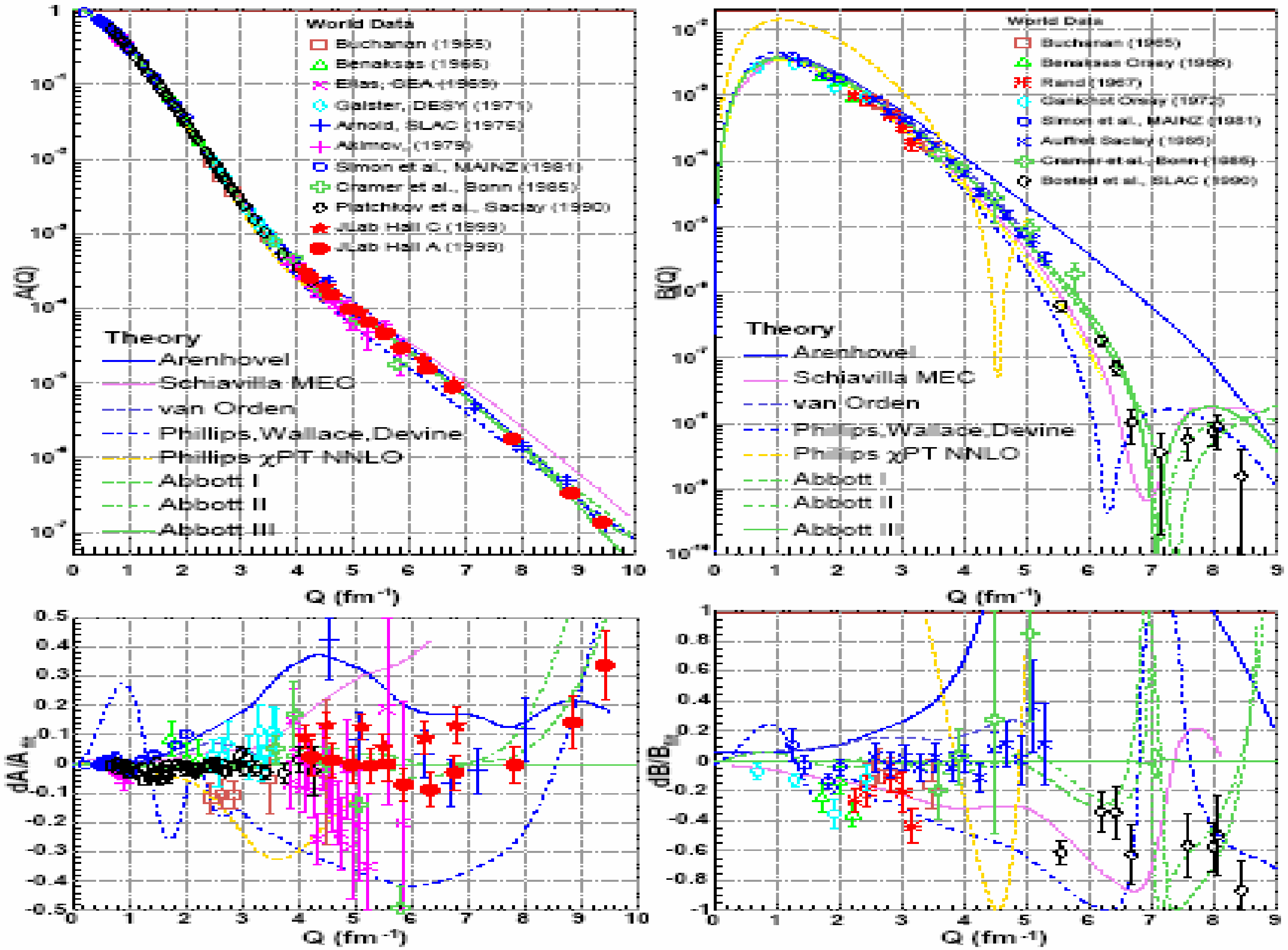
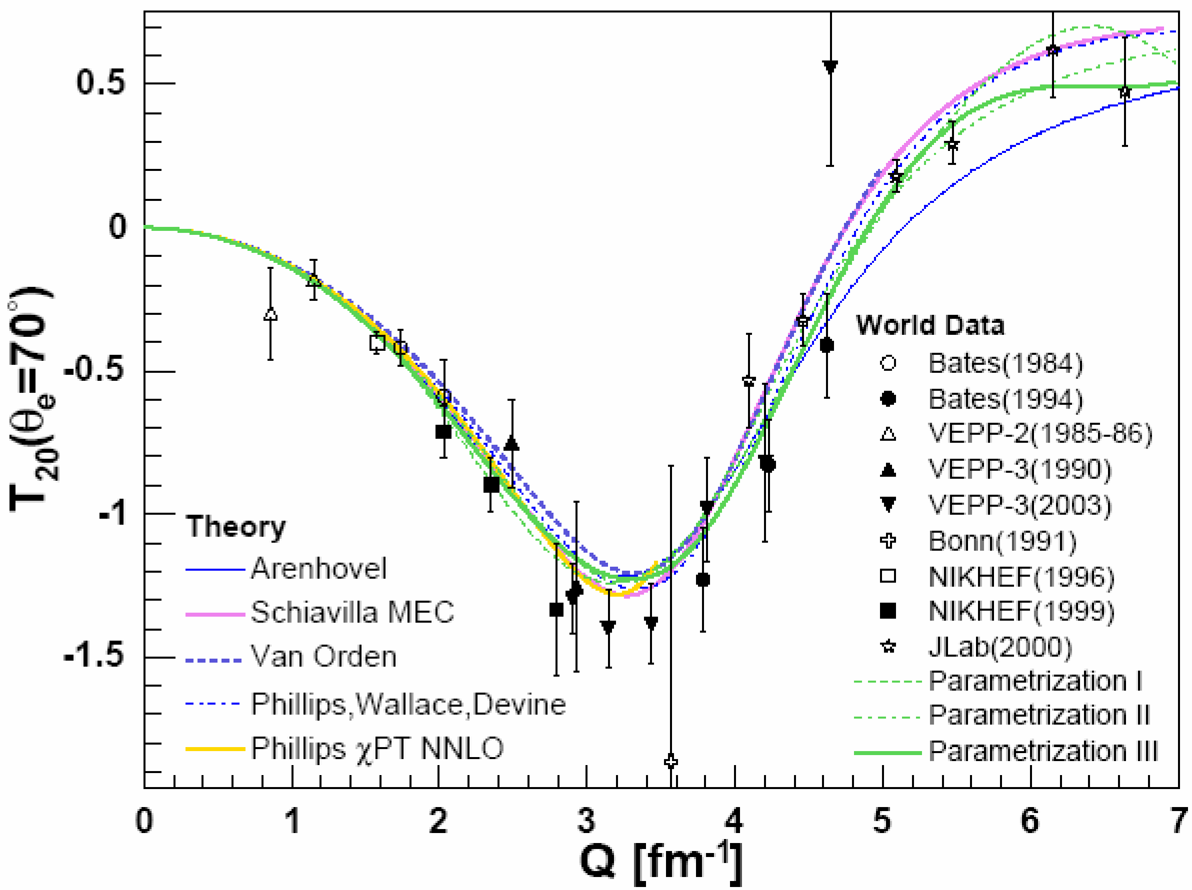
Non-relativistic calculations with MEC, RC



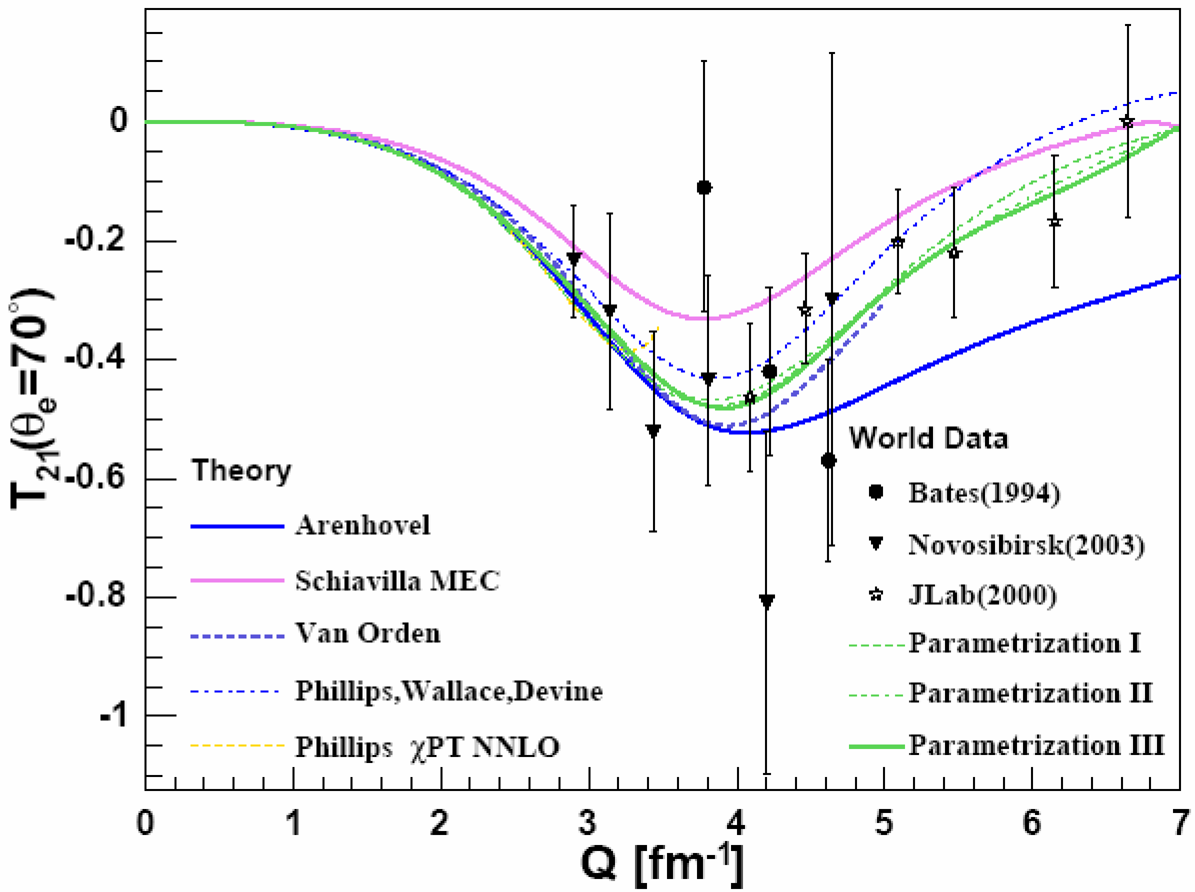
Non-relativistic calculations with MEC, RC




World Data
Buchanan (1965)
Benaksas (1966)
Elias, CEA (1969)
Galster, DESY (1971)
Arnold, SLAC (1975)
Akimov, (1979)
Simon et al., MAINZ (1981)
Cramer et al., Bonn (1985)
Platchkov et al., Saclay (1990)
JLab Hall C (1999)
JLab Hall A (1999)
A(Q)
Theory
Arenhovel
Schiavilla MEC
van Orden
Phillips,Wallace,Devine
Phillips χPT NNLO
Abbott I
Abbott II
Abbott III
Q (fm⁻¹)
World Data
Buchanan (1965)
Benaksas Orsay (1966)
Rand (1967)
Ganichot Orsay (1972)
Simon et al., MAINZ (1981)
Auffret Saclay (1985)
Cramer et al., Bonn (1985)
Bosted et al., SLAC (1990)
B(Q)
dA/A
dB/B



$T_{20}(\theta_e=70^\circ)$
Q [fm^{-1}]
0.5
0
-0.5
-1
-1.5
0
1
2
3
4
5
6
7
Theory
Arenhovel
Schiavilla MEC
Van Orden
Phillips,Wallace,Devine
Phillips χPT NNLO
World Data
Bates(1984)
Bates(1994)
VEPP-2(1985-86)
VEPP-3(1990)
VEPP-3(2003)
Bonn(1991)
NIKHEF(1996)
NIKHEF(1999)
JLab(2000)
Parametrization I
Parametrization II
Parametrization III



$T_{21}(\theta_e=70^{\circ})$
Q [fm^{-1}]
0
-0.2
-0.4
-0.6
-0.8
-1
0 1 2 3 4 5 6 7
Theory
Arenhovel
Schiavilla MEC
Van Orden
Phillips,Wallace,Devine
Phillips χPT NNLO
World Data
Bates(1994)
Novosibirsk(2003)
JLab(2000)
Parametrization I
Parametrization II
Parametrization III

Definition of T_{20R}

We can rewrite T_{20} in the following manner:

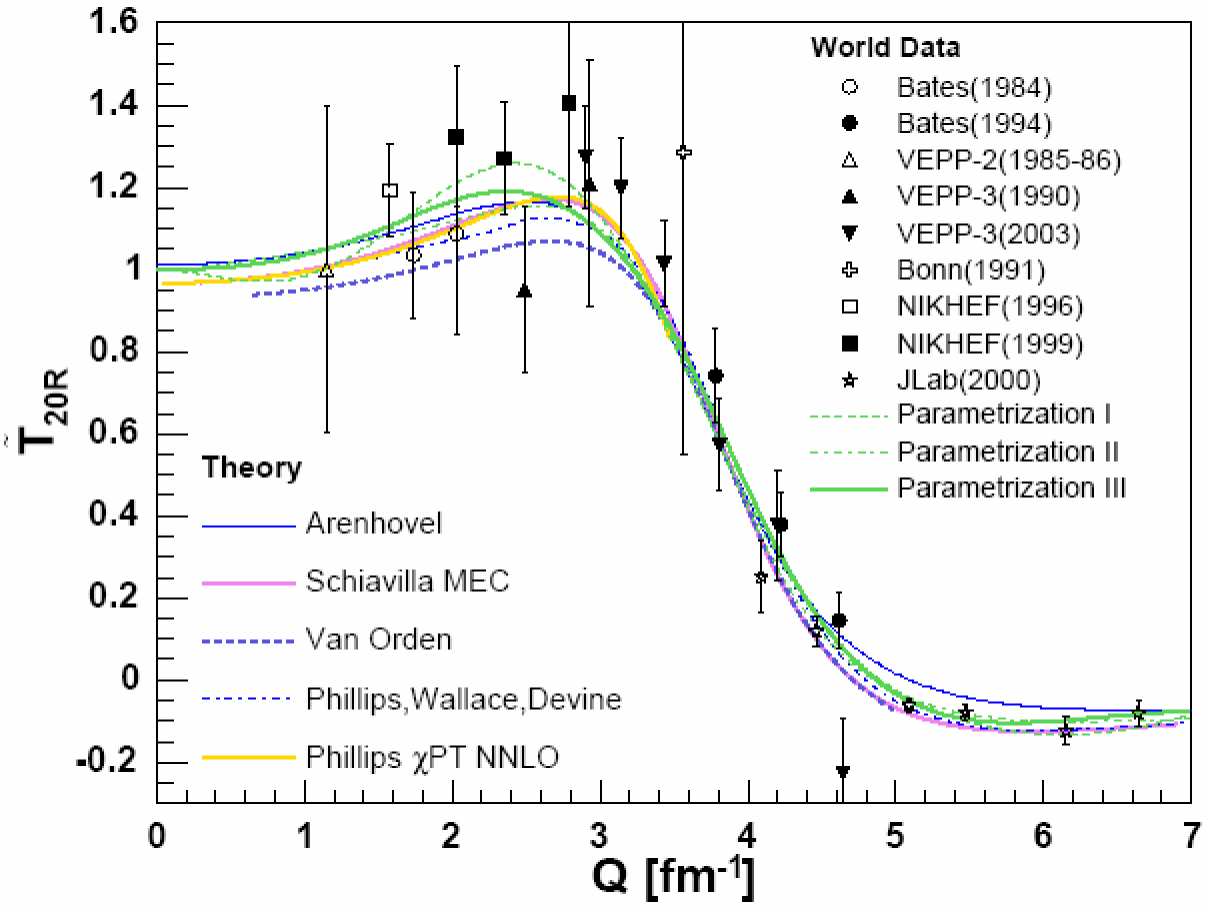
$$T_{20} = -\frac{(4Y + 2Y^2 + X^2/2)}{\sqrt{2}(1 + 2Y^2 + 2X^2)}$$

where $Y = \frac{2\eta G_Q}{3G_C}$

and $X = X(\frac{G_M}{G_C})$ **… some complicated function**

We now get $\tilde{T}_{20} = -\frac{\sqrt{2}Y(2+Y)}{1+2Y^2}$ **by subtracting *X* dependence**

Finally define $T_{20R} = -\frac{3\tilde{T}_{20}}{\sqrt{2}Q^2 Q_d}$ **which → 1 as Q^2 → 0 if Q_d is in units of M^2 *… Garco'n and van Orden***



World Data
Bates(1984)
Bates(1994)
VEPP-2(1985-86)
VEPP-3(1990)
VEPP-3(2003)
Bonn(1991)
NIKHEF(1996)
NIKHEF(1999)
JLab(2000)
Parametrization I
Parametrization II
Parametrization III
Theory
Arenhovel
Schiavilla MEC
Van Orden
Phillips,Wallace,Devine
Phillips χPT NNLO
$\tilde{T}_{20R}$
Q [fm^{-1}]
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
0
1
2
3
4
5
6
7

T_{20} and Form Factors

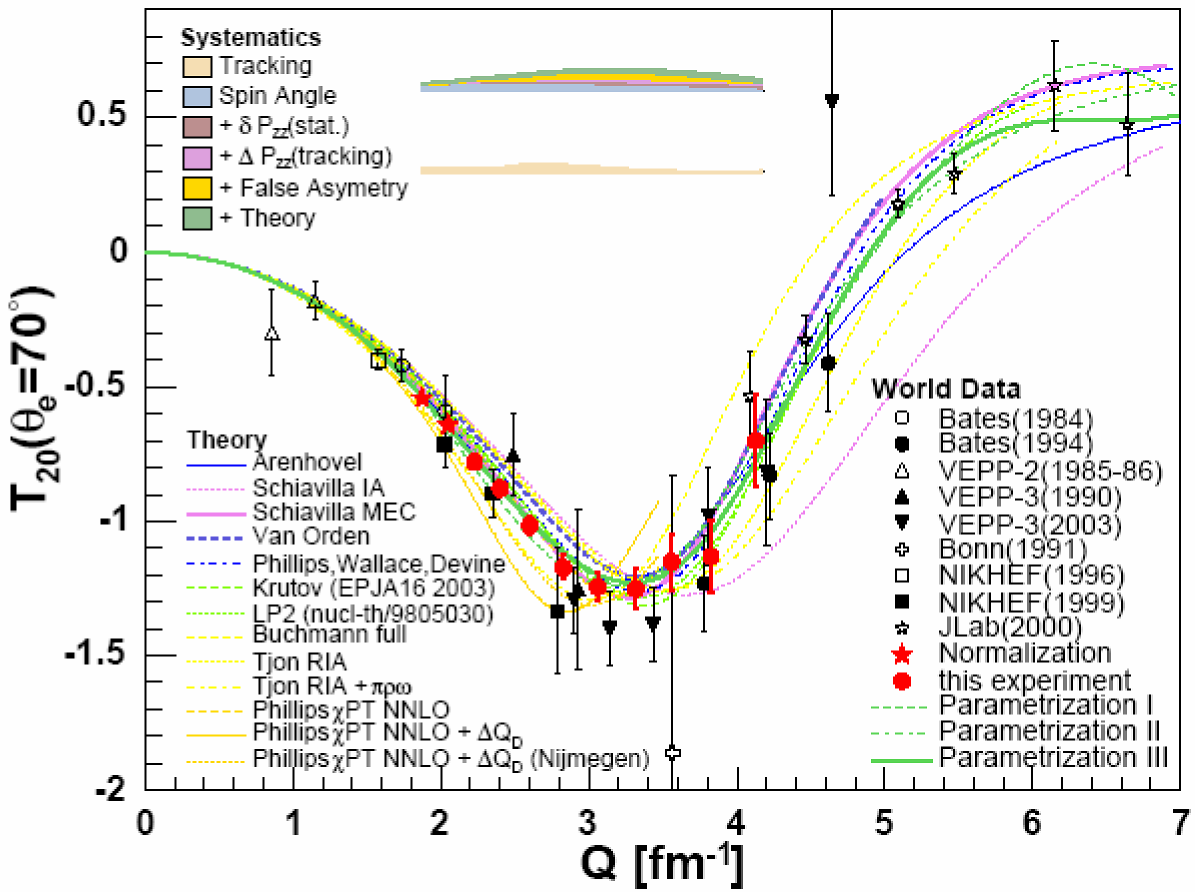
- Observe 2 Asymmetries simultaneously in parallel and perpendicular kinematics
- Use world data to subtract T_{22} contributions:
  a few % of total Asymmetries.

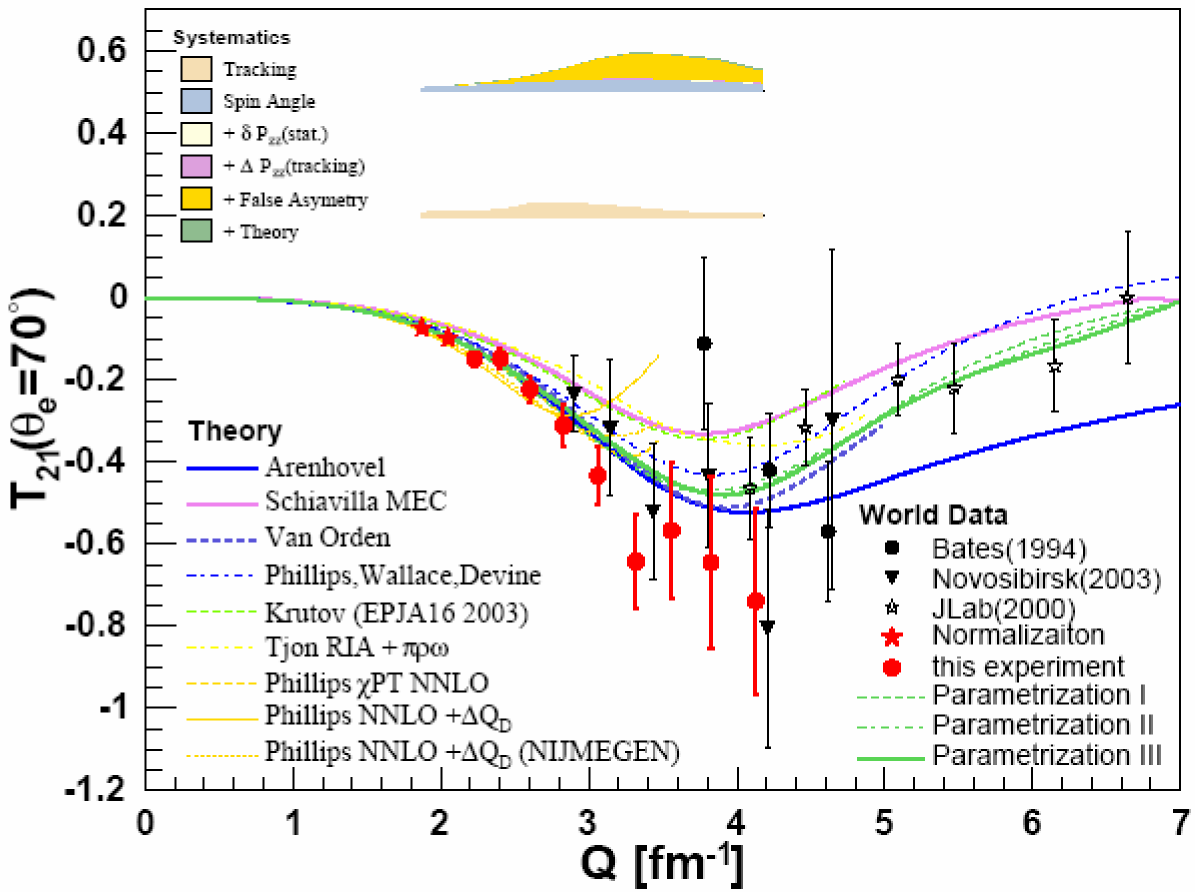
$$\mathbf{A}^* = \frac{3\cos^2\theta_d^* - 1}{2} T_{20} - \sqrt{\frac{3}{2}} \sin 2\theta_d^* \cos\phi_d^* T_{21}$$

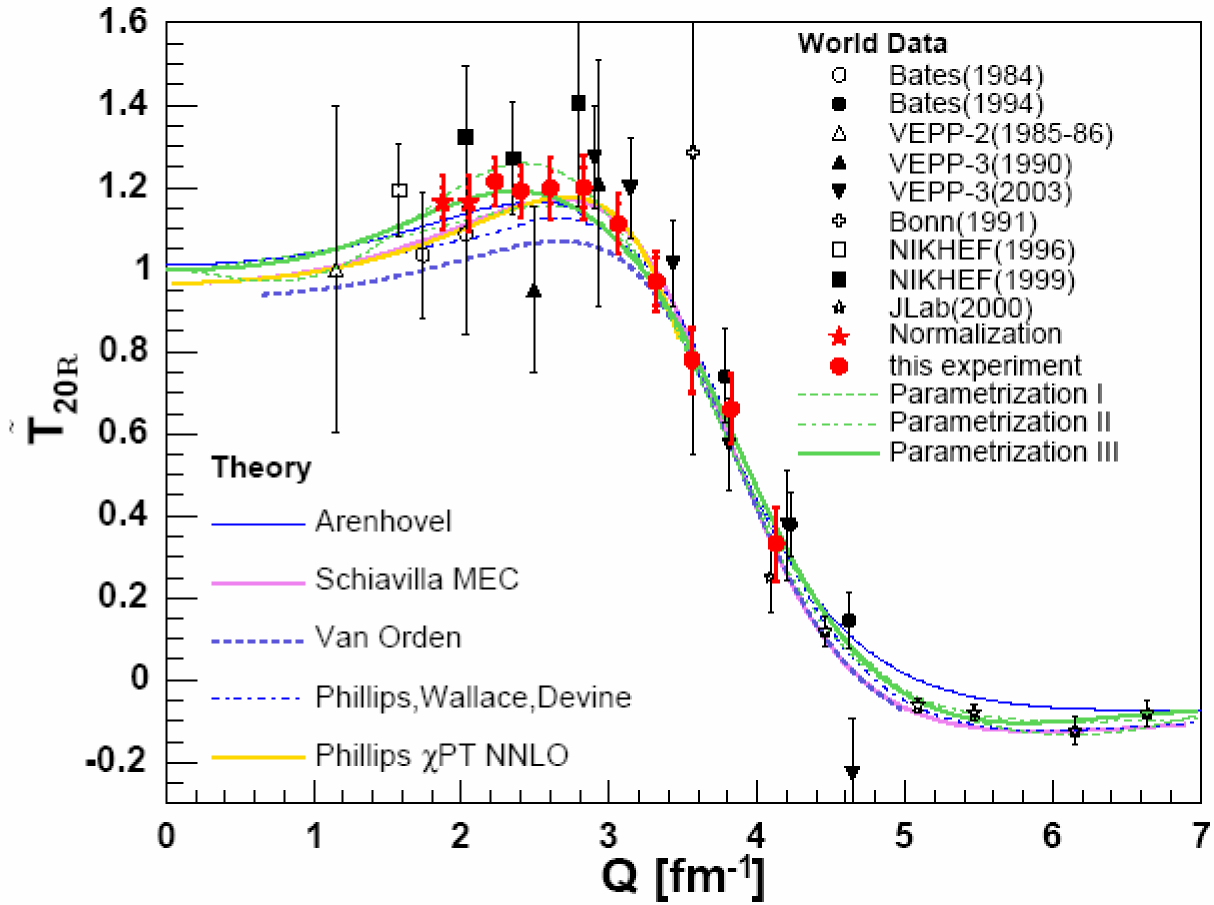
- Solve for T_{20} and $T_{21,}$ from $A_{\parallel}$ and $A_{\perp}$
- Or use world data to subtract T_{21} contributions
- Use world data of A(Q), and BLAST Asymmetries, use world data for G_M contributions, least square fit for G_C(Q) and G_Q(Q),

$$\chi^2 = (A - A(Q))^2/\delta A^2 + (Asym - Asym(Q))^2/\delta Asym^2$$

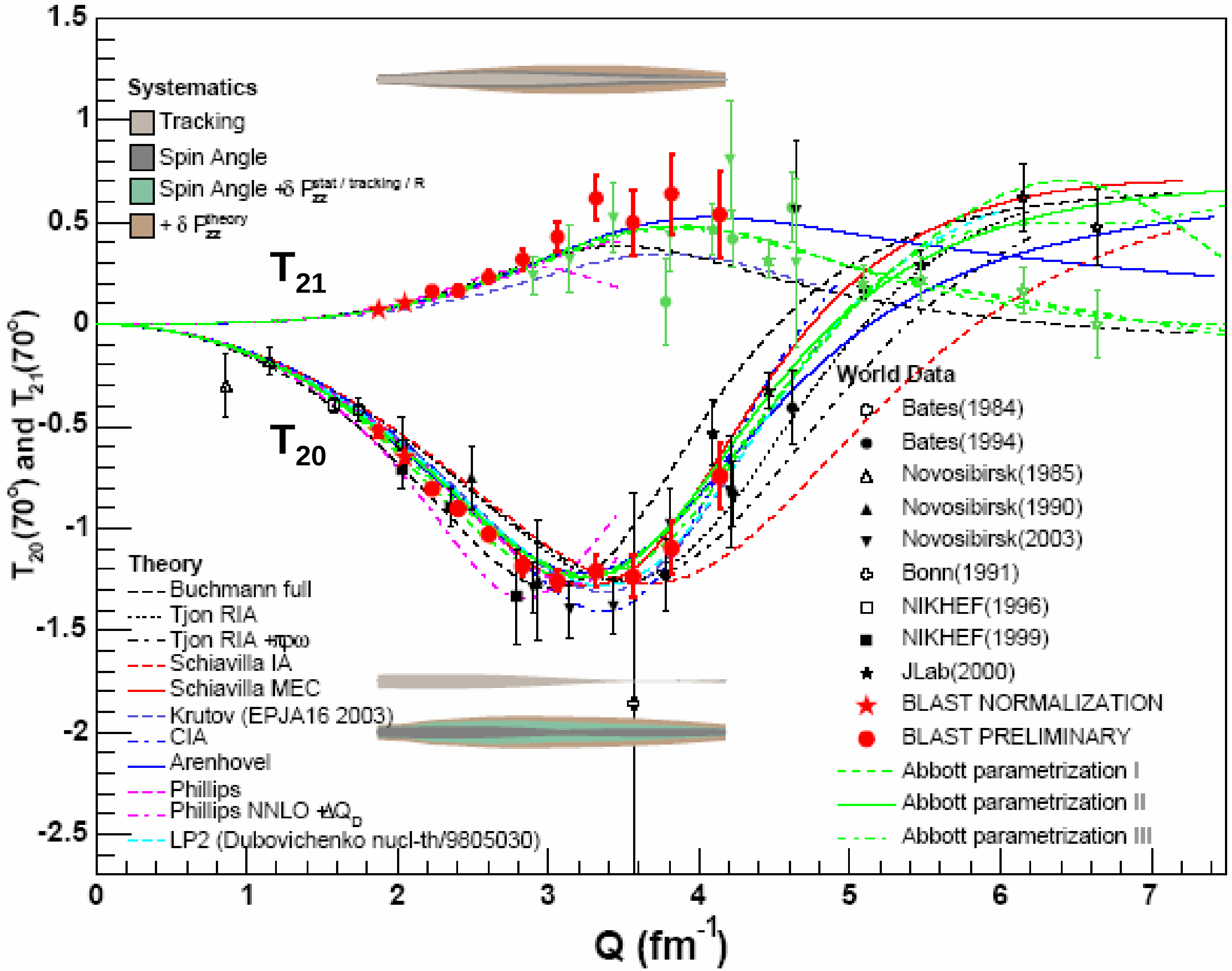
- Use BLAST data and world data of cross sections and polarized observables, refit Abbott's Parameterizations
  Parameterization I finished


Systematics
Tracking
Spin Angle
+ δ P_{zz}(stat.)
+ Δ P_{zz}(tracking)
+ False Asymetry
+ Theory
Theory
Arenhovel
Schiavilla IA
Schiavilla MEC
Van Orden
Phillips,Wallace,Devine
Krutov (EPJA16 2003)
LP2 (nucl-th/9805030)
Buchmann full
Tjon RIA
Tjon RIA + πρω
Phillips χPT NNLO
Phillips χPT NNLO + ΔQ_D
Phillips χPT NNLO + ΔQ_D (Nijmegen)
World Data
Bates(1984)
Bates(1994)
VEPP-2(1985-86)
VEPP-3(1990)
VEPP-3(2003)
Bonn(1991)
NIKHEF(1996)
NIKHEF(1999)
JLab(2000)
Normalization
this experiment
Parametrization I
Parametrization II
Parametrization III
$T_{20}(\theta_e=70^\circ)$
Q [fm^{-1}]
0.5
0
-0.5
-1
-1.5
-2
0
1
2
3
4
5
6
7

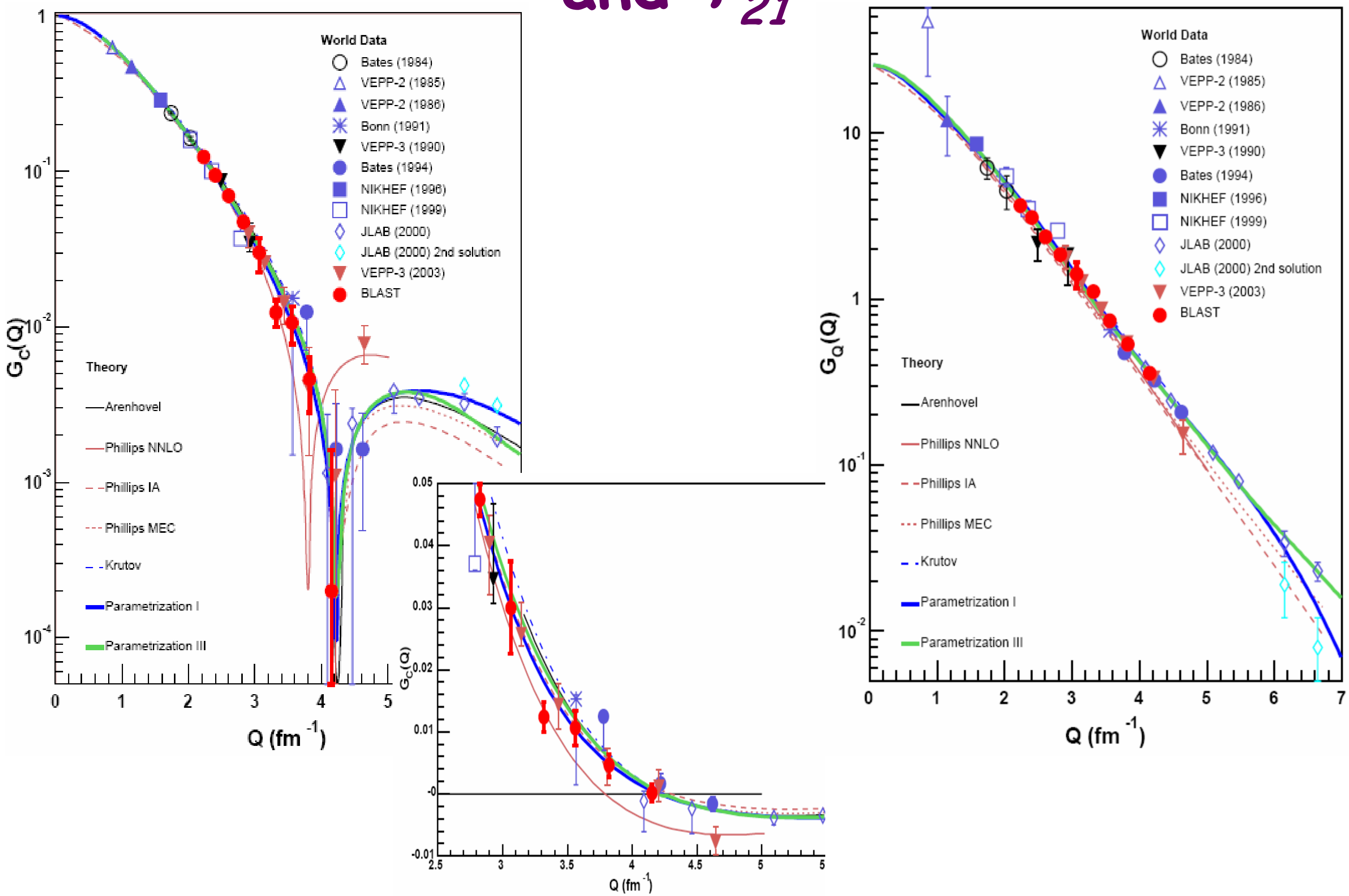

Systematics
Tracking
Spin Angle
+ δ P_zz(stat.)
+ Δ P_zz(tracking)
+ False Asymetry
+ Theory
Theory
Arenhovel
Schiavilla MEC
Van Orden
Phillips,Wallace,Devine
Krutov (EPJA16 2003)
Tjon RIA + πρω
Phillips χPT NNLO
Phillips NNLO +ΔQ_D
Phillips NNLO +ΔQ_D (NIJMEGEN)
World Data
Bates(1994)
Novosibirsk(2003)
JLab(2000)
Normalizaiton
this experiment
Parametrization I
Parametrization II
Parametrization III
T_21(θ_e=70°)
Q [fm^-1]
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-1.2
0
1
2
3
4
5
6
7


World Data
Bates(1984)
Bates(1994)
VEPP-2(1985-86)
VEPP-3(1990)
VEPP-3(2003)
Bonn(1991)
NIKHEF(1996)
NIKHEF(1999)
JLab(2000)
Normalization
this experiment
Parametrization I
Parametrization II
Parametrization III
Theory
Arenhovel
Schiavilla MEC
Van Orden
Phillips,Wallace,Devine
Phillips χPT NNLO
$\tilde{T}_{20R}$
Q [fm^{-1}]
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
0
1
2
3
4
5
6
7

Results: T_{20} and T_{21}



Results: Form Factors from *A*, *B*, T_{20}, and T_{21}



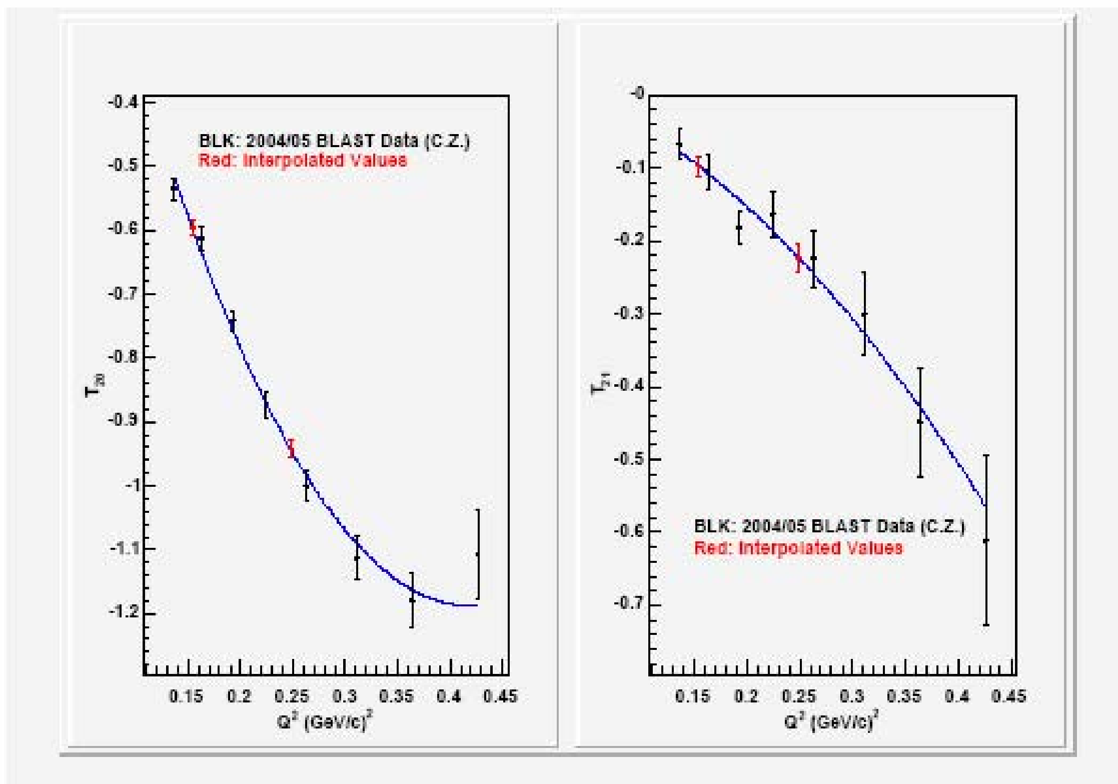
Extracting the Form Factors … including G_M

- We have T_{20}, T_{21}, and T_{11}^e from BLAST.
- Use only $A(Q^2)$ from world data

$$
\begin{aligned}
T_{11}^e &= \sqrt{\tfrac{3}{2}}\tfrac{1}{S}\tfrac{4}{3}[\tau(1+\tau)]^{1/2}G_M(G_C+\tfrac{\tau}{3}G_Q)tan\tfrac{\theta_e}{2} \\
T_{20} &= -\sqrt{2}\tfrac{1}{S}\tau\left(\tfrac{4}{3}G_CG_Q+\tfrac{4}{9}G_Q^2+\tfrac{1}{6}(1+(\tau+1)tan^2(\theta_e/2))G_M^2\right) \\
T_{21} &= -\tfrac{2}{\sqrt{3}}\tfrac{1}{S}\tau\left(\tau+\tau^2 sin^2(\theta_e/2)\right)^{1/2}G_MG_Q sec\tfrac{\theta_e}{2} \\
A(Q^2) &= G_C^2(Q^2)+\tfrac{8}{9}\tau^2G_Q^2(Q^2)+\tfrac{2}{3}\tau G_M^2(Q^2)
\end{aligned}
$$

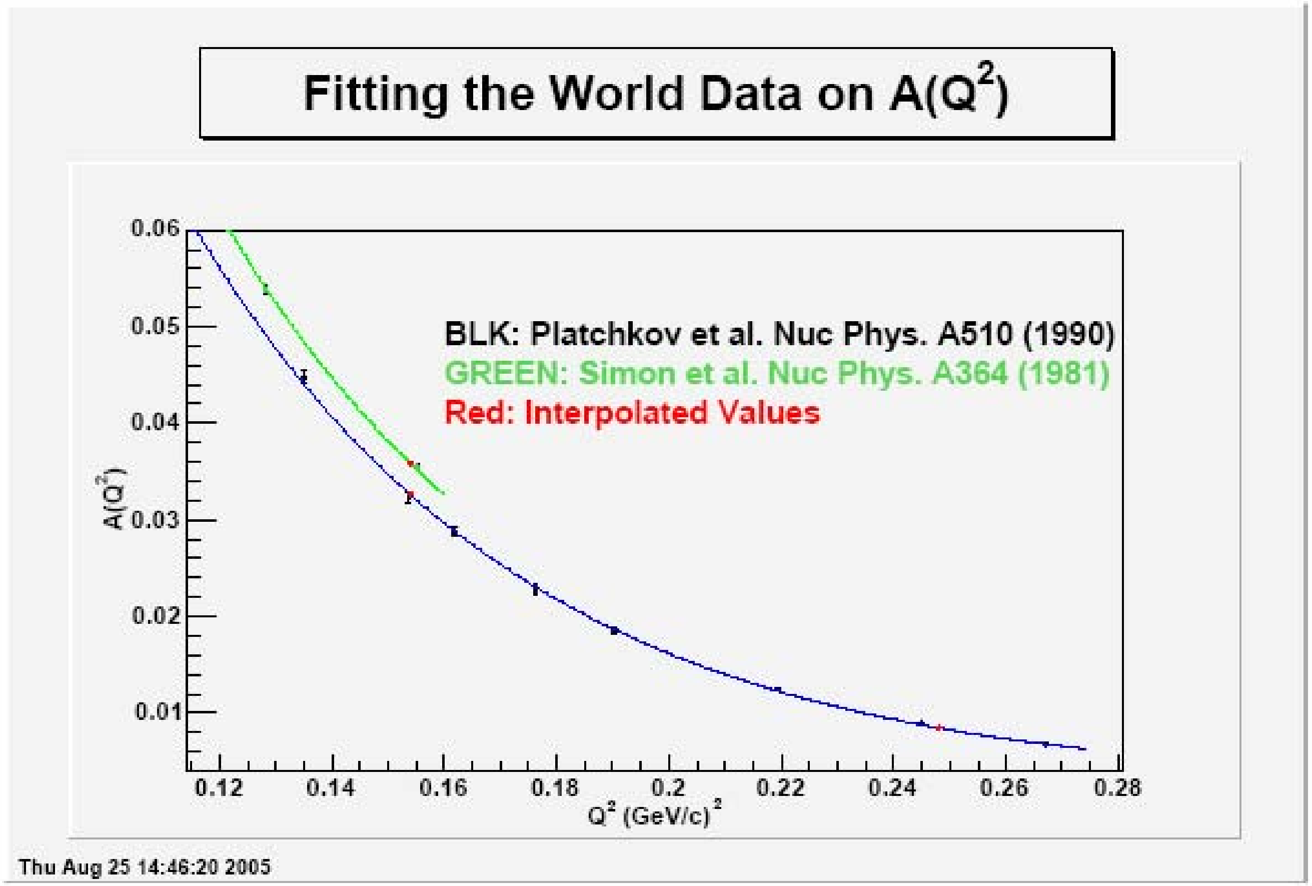
- 4 equations - 3 parameters $\rightarrow$ 1 D.O.F.

Fitting Chi's T_{20} and T_{21}

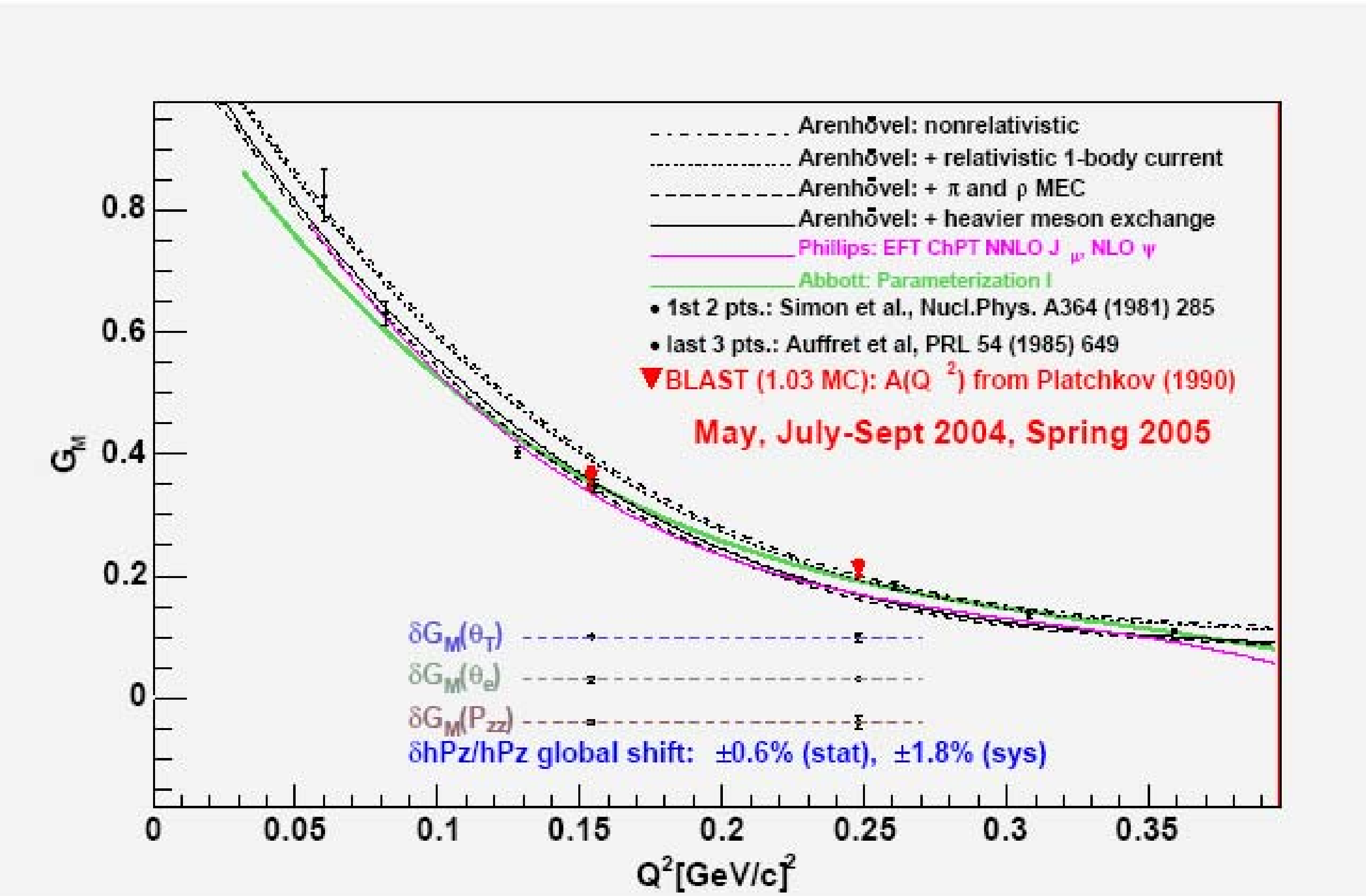


- Courtesy of Chi Zhang, MIT

Saclay & Mainz Discrepo!



G_M with Saclay $A(Q^2)$



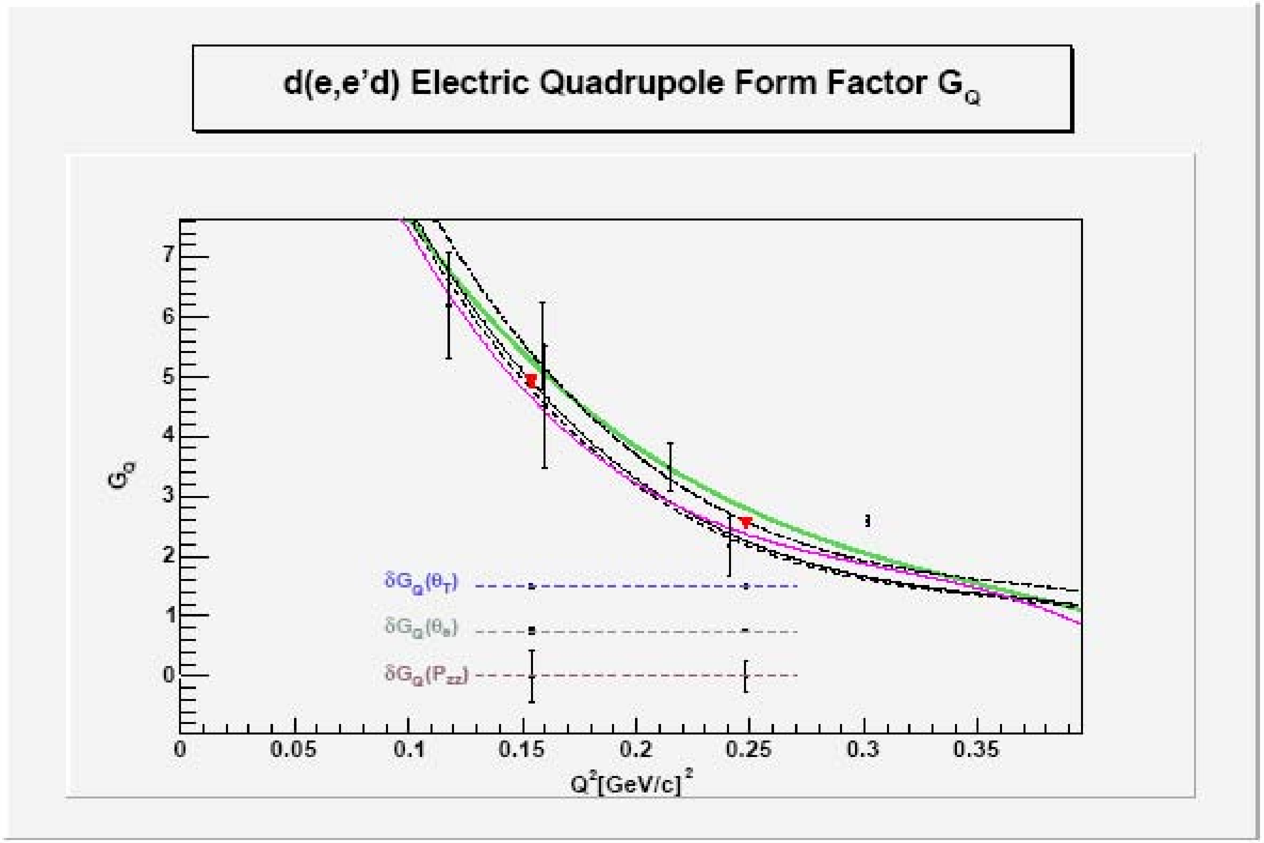
Preliminary Results Summary

$$Q^2 = 0.154 \; [GeV/c]^2$$
$$T^e_{11} = 0.0599 \pm 0.0029$$
$$G_M = 0.3615 \pm 0.0172 \text{ (Saclay } A(Q^2))$$
$$G_M = 0.3787 \pm 0.0180 \text{ (Mainz } A(Q^2))$$

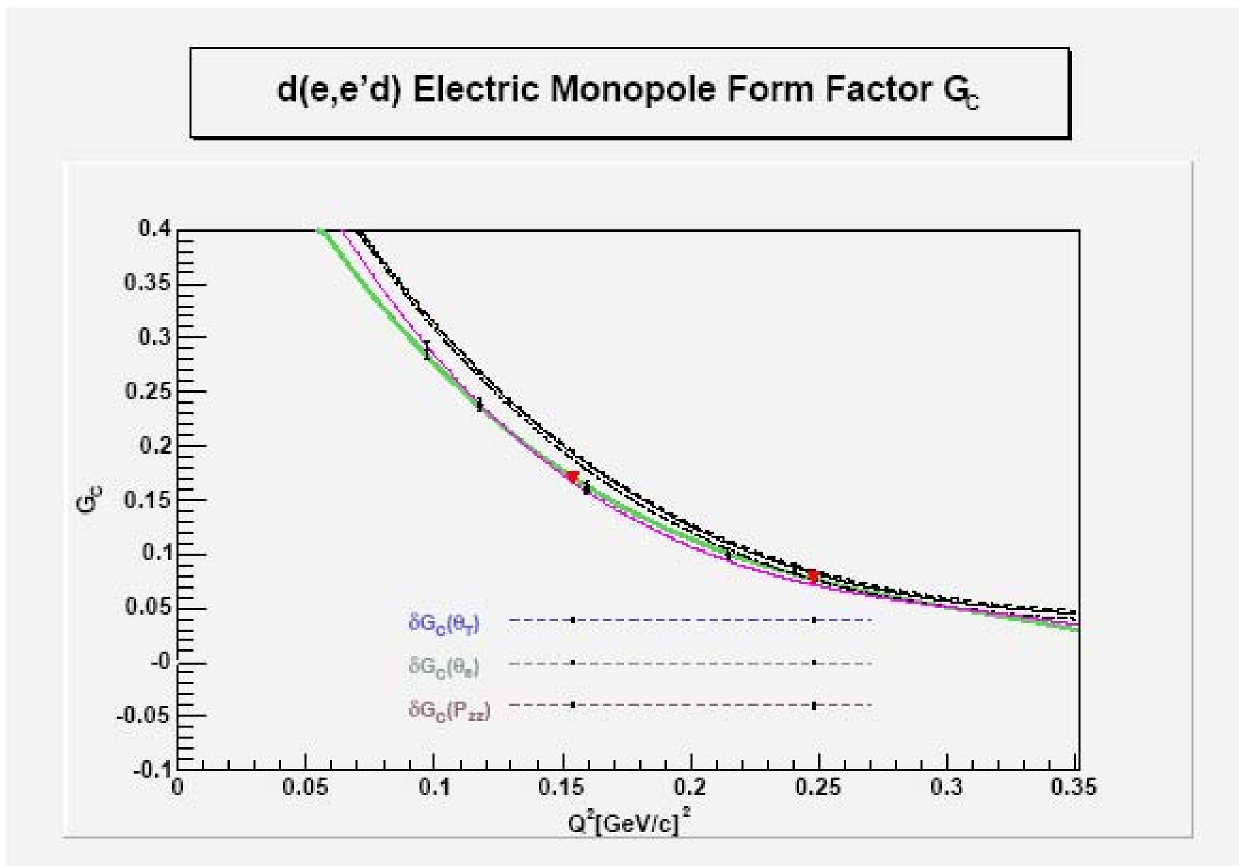
$$Q^2 = 0.248 \; [GeV/c]^2$$
$$T^e_{11} = 0.1035 \pm 0.0066$$
$$G_M = 0.2119 \pm 0.0120$$

- First measurement of T^e_{10} and T^e_{11}
- Unique measurement of G_M from spin observables
- Motivation for new $A(Q^2)$ data at low Q^2

G_Q with Saclay $A(Q^2)$



G_C with Saclay $A(Q^2)$



Conclusions

- First measurement of T^{e}_{11}
- World class measurement of T_{20}
- Sensitive to the ***D-state*** of the deuteron
- Sensitive to ***MEC, IC, RC***
- Extraction of G_C , G_Q , G_M